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INTRODUCTION TO THE SUBMARINE DAMAGE MECHANISMS PROJECT

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This report describes a statistical methodology that forms a key part of an interdisciplinary approach to damage prediction. The principal problem addressed is that of predicting submarine hull rupture caused by an underwater explosion. The statistical approach is compared with two other current approaches to the problem. The statistical approach is shown to be a synthesis and extension of these more deterministic alternatives. The report discusses the role of deterministic physical models in response prediction and how powerful deterministic computer codes can be utilized in constructing probability models for dependent, multivariate damage responses of interest. It describes how the statistical approach provides an essential framework within which issues such as prediction accuracy, model validation, and decision making can be qualitatively and objectively addressed.

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FOREWORD

This report is intended to be an easily understandable account of the Submarine Damage Mechanisms Project and, in particular, of a statistical methodology for modeling submarine hull rupture resulting from an underwater explosion. The project has been a collaborative effort conducted over several years by the Naval Surface Warfare Center at White Oak, MD, the Massachusetts Institute of Technology, and SRI International. The work has been sponsored by the Office of Naval Research Explosives and Undersea Warheads Technology Program and the Submarine Survivability Program.

Earlier versions of this report were distributed to a number of individuals for review. For their interest and feedback the author is grateful to Dr. Edgar Cohen of the Naval Surface Warfare Center Dahlgren Division, White Oak; to Mr. Robert Barash, Mr. Edward Johnson, Mr. Robert Kavetsky, Mr. John Koenig, Mr. Hans Mair, and Dr. Minos Moussouros of the Indian Head Division, Naval Surface Warfare Center, White Oak; to Mr. John Barnett of the Naval Command, Control and Ocean Surveillance Center; to Dr. Michelle Hoo Fatt, Professor Frank McClintock, and Professor Tomasz Wierzbicki of the Massachusetts Institute of Technology; to Dr. Jacques Giovanola of SRI International; to Dr. Terry Klopsic, Mr. Bill Baker, and Dr. Stephen Wilkerson of the Army Research Laboratory, Aberdeen; to Dr. Judah Goldwasser of the Office of Naval Research; and to Professor Nozer Singpurwalla of the George Washington University. The author is particularly indebted to those who made valiant efforts to follow the thread of the technical details, made constructive comments, and freely expressed both their opinions and confusions, before many of the loose ends were tied together. Based on this input, the report was substantially reworked with much new material added to enhance clarity and much of the rhetoric deleted. The contributions of these individuals significantly improved the quality of this report and the communication of the ideas. The author is most appreciative.

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The Submarine Damage Mechanisms (SDM) Project represents a continuation of work begun in the mid 1970s concerned with the development of rigorous methods for predicting the effects of explosions upon ships and submarines and for assessing the performance of ASW and ASUW* weapon systems. The approach has evolved under a variety of sponsors, principally the Office of Naval Research (and formerly the Office of Naval Technology [ONT]) and the Defense Nuclear Agency. Earlier applications have included submarine hull rupture by low yield nuclear weapons, ship whipping damage, ship sinking due to loss of bottom section modulus, internal equipment damage, and the failure of major ship systems. Each application has extended and broadened the theory and has promoted a more fundamental understanding of the damage problem in general. Under the SDM Project the approach has developed into a general methodology that should be useful for solving many problems of both military and general interest. In addition to describing the SDM Project, this paper attempts to state in clear language the methodology and many of the underlying ideas.

The application we now call the SDM Project grew out of a 1984 ONT-sponsored effort at the Naval Surface Warfare Center (NSWC) known as the Target Response Task. The objective of the Target Response Task was to develop computational methods for predicting hull rupture by underwater warheads when used against adversary submarines. Areas of focus included (1) the understanding and improvement of existing empirical damage rules through the use of finite element modeling techniques, (2) the development of constitutive models for new materials, and (3) the development of improved failure criteria. In FY 87 the author assumed management of the task and extensively refocussed the effort in light of his earlier work on the hull rupture problem.

The principal objective of the SDM Project remains that of predicting the rupture of a submarine hull as a result of an underwater explosion, but in addition we address a number of broader issues. The project now consists of three subtasks. Task 1, referred to as the Hull Rupture Modeling Task, seeks to integrate the disciplines of

* ASW and ASUW are abbreviations for anti-submarine warfare and anti-surface ship warfare, respectively.

structural mechanics and fracture mechanics with empirical test data within a statistical framework to predict the probability of breaching the pressure hull, i.e., of producing a crack through the thickness of the pressure hull (a "through-crack"). Task 2, added in 1994 and known as the Catastrophic Failure Task, is regarded as an extension of Task 1 and is concerned with the extension of the length of the through-crack to large values relative to hull dimensions. Task 3, also added to the project in 1994, is referred to as the Concept Assessment Methodology Task. Its objective is to develop a method for comparing the performances of different warhead design concepts and selecting between them. Under Task 3 decision theory and operations research techniques are used to bring other considerations, such as cost and development risk, into the selection process in addition to considerations of warhead performance. The overall objective of the SDM Project is to develop computational and analytical tools that the Navy will find useful for making both warhead and submarine design decisions, and for establishing effective operational policies for their use.

Tasks 1 and 3, in particular, reflect the inclusion of probabilistic and statistical techniques in the analyses. The use of nondeterministic modeling techniques, in addition to engineering and physics response models, in fact, distinguishes the SDM Project from other more traditional attempts to analyze the explosion damage problem. Historically, the prediction of submarine hull rupture has been approached by the Navy in two distinctly different ways. For discussion purposes, we will refer to these as the semi-empirical and quasi-deterministic approaches.

The earliest prediction methods, many of which are still being used, are based upon semi-empirical approaches. Prior to and throughout the 1970s, most hull response prediction techniques ("damage rules") used by the Navy were developed by fitting hull deflection data, obtained from reduced-scale experimental tests, to formulas involving structural design and test conditions, and often including variables motivated by simplified response analyses. Empirical criteria for rupture were expressed in terms of critical hull deflections.

More recent analysis methods predict structural responses by solving the fundamental equations of continuum mechanics. We shall characterize such methods as quasi-deterministic, a term coined to connote the notion that the models are basically deterministic but with input from empirical tests in which scatter was minimized. This approach has been made possible by the advent of modern computers, and it has become widely followed due to the emergence of general purpose structural response

codes. Criteria for the onset and development of ductile failures are typically expressed in these codes in terms of critical values of local variables that depend upon components of the strain and/or stress tensors, such as equivalent plastic strains and expressions for void volume fractions. Under this approach empirical information, usually obtained by laboratory testing of small material specimens, is employed in the formulation of the constitutive and failure models and the determination of failure thresholds.

The methodology being developed under the SDM Project can be viewed as a synthesis of the semi-empirical and the quasi-deterministic approaches. We refer to it as a statistical approach. Like the semi-empirical approaches, we attempt to model observed experimental responses for the system of interest, in particular fracture onset and crack length responses, and then use the model to predict the occurrence and magnitude of the responses under arbitrary conditions. Like the quasi-deterministic methods, physics and engineering models are used within the statistical approach to represent the systematic mean behavior of responses as functions of the problem conditions. But in addition to these, we include models for the random behavior, which is evident in the test data, on top of the systematic behavior. This results in a more complete model for the responses and a model that permits more objective investigations of important issues which often elude quasi-deterministic analyses alone, such as model validation and prediction accuracy. The statistical approach we are developing represents an attempt to overcome (1) the criticism of the semi-empirical models as lacking in enough of the underlying physics and (2) the difficulties encountered by quasi-deterministic models in matching the experimental data. It also represents an attempt to more fully utilize the Navy experimental database of explosive tests against stiffened shells.

In this report we present the hull rupture prediction problem from a statistical viewpoint. Because scatter is unavoidable in structural test data and always evident upon close examination, rigorous models for the observed behavior are necessarily probabilistic. Interdisciplinary statistical models are needed to characterize the response probabilities when the magnitude of the response scatter is significantly large and when prediction errors are of high consequence. It is argued that the development of such models is essential for the resolution of the hull rupture prediction problem in both lethality and survivability contexts, and for the resolution of many other problems of military interest. We will attempt to show the many benefits that are derived by casting the problem in such a manner.

The report is organized as follows. The next section characterizes the submarine hull rupture prediction problem from a nondeterministic viewpoint to set the context of the discussion. Then, brief accounts of the semi-empirical and quasi-deterministic approaches are presented which describe their usefulness but also point out various difficulties encountered when these approaches are applied to the problem of system damage prediction. After these, we then present the statistical approach and describe in simple language its basic concepts and elements that can be used to model responses more generally. This is followed by a description of the SDM Project which illustrates its interdisciplinary nature and outlines how the different disciplines can be effectively and naturally combined to produce a solution for the hull rupture prediction problem. The concluding section addresses some broader modeling issues. Subheadings are employed within the longer sections.

CHARACTERIZATION AND FORMULATION OF PROBLEM

Damage Problems, Navy Needs, and Decisions:

When an explosive charge detonates near the hull of a submerged submarine, intense pressures are generated over a somewhat localized area of the pressure hull which cause the hull to deform inward. Circular stiffening rings, cut from flat plates and welded at regular intervals along the cylindrical pressure hull for the purpose of reinforcement, will initially resist the inward motion of the hull at their points of attachment. Large stresses and strains will particularly develop in the vicinities of the stiffener-to-hull weldments in both the hull and stiffener plates. Although stiffening rings typically have top flanges attached to discourage out-of-plane motion, the stiffeners will distort and buckle unstably if the pressures are of sufficient magnitude and duration, and especially when significant stiffener side loads have been developed. For submarine designs where the rings have been welded on the exterior side of the pressure hull, a stiffener will often respond in an alternative mode and fracture along its welded base. Both mechanisms, buckling of the stiffener and its detachment by fracturing, will cause a redistribution of the stress and elastic strain fields in the structure. Generally, these will be lowered in the hull plating near the affected stiffener, but increased in the vicinities of neighboring stiffeners. The response of the submarine is thus governed by a complicated interplay of response mechanisms and load transfers that may result in the development of local strains and stresses in the plating of the pressure hull of sufficient magnitude to cause hull rupture. Because submarine structural steels are tough and highly ductile, large plastic deformations of the hull and stiffeners will occur prior to hull rupture.

Because of the diversity and complexity of the responses produced in a submarine by an explosion and the various contexts in which the response problem is important, the problem of predicting explosion damage to submarines means different things to different people. Perhaps, initially, there appear to be many different unrelated problems. Upon closer inspection, however, damage problems of diverse descriptions are found to have much in common.

The performance and reliability of weapon systems is sometimes described as the "lethality problem"; the damage resistance of the submarine target directly affects the performance or lethality of the anti-submarine weapon system. Closely related is the need to achieve explosion-resistant designs for U.S. submarines. In this defensive context the problem of predicting submarine damage is often referred to as the submarine "survivability problem." A part of the survivability problem concerns the formulation and selection of tough, fracture-resistant hull materials. Materials scientists use the term "damage" in a specialized sense to refer to microstructural response processes, such as the growth of microvoids or microcracks, that degrade the local strength properties of a structural material. Using more broadly defined terms, military planners and evaluators may want to minimize or maximize damage to "our" or "their" submarines, respectively, in specific encounter scenarios. In all contexts the terms *damage* and *failure* connote a similar idea: a deleterious response of the submarine in some specific sense that limits the submarine's performance capabilities. This similarity of concepts translates into similarities in the statistical aspects of mathematical models for damage prediction.

Explosion damage problems are closely associated with the need for making decisions, often, if not always, in the presence of uncertainties. Usually these are system design and acquisition decisions or decisions regarding the use and management of particular systems. It is preferable, obviously, that these decisions be made objectively and in a scientific manner. Decision theory is a well-developed scientific discipline that relies heavily on statistical methods to model the uncertainties that complicate the decision-making process. While objective methods for measuring and predicting the responses of well-defined structures to well-defined loads have been appropriately funded and extensively researched, it appears that relatively little attention, by comparison, has been given to the task of translating those results into objective decision making. We propose that statistical methods combined with the deterministic methods of physics and engineering can facilitate that linkage. But these methods do not now exist and need to be developed.

In a 1961 report, Dr. William Murray of the David Taylor Model Basin wrote, "It is believed that the time is ripe for prediction methods to be organized in such a way that the basic methods and damage classification employed by all groups which generate inputs to the same overall military problem will be rendered compatible.... The researcher recognizes that his problem is to some extent statistical.... The technical

problem may appear simple within the framework of the overall task of the military evaluator. However, the research scientist, who must play the role of predictor in order to supply the answer, has certain limitations. These can only be conveyed to the evaluator by an assessment of the reliability of the weapon-effects predictions.... Methods of preparing answers must be formalized." Later, in a section outlining his approach Murray said, "A given attack severity will cause shipboard damage which cannot be precisely predicted in any one test. The reason is that damage (hull rupture, equipment failure, personnel injury) depends on many details which the predictor cannot possibly know, even in a carefully controlled test. This type of ignorance is handled by a statistical treatment of available experimental data.... Existing experimental data would be utilized to relate attack severity to the probability of impairment, see Figure 10." Murray's Figure 10 was an illustration of impairment probability curves as functions of an attack severity quantity for various categories or levels of impairment.[1]

Murray's insights into the problems of response prediction and military decision making, which are unfortunately buried in a classified report, remain valid today. Since that time decision theoretic and statistical methods have advanced considerably, as have the deterministic methods of physics and engineering. The time is now even more ripe for creating a comprehensive approach.

Categories and Types of Uncertainties:

A basic requirement for developing effective solutions to complex response and decision problems is to distinguish between the types of information and uncertainties involved. Some of the information out of which a solution is to be developed will be "hard" information (i.e., information that we are prepared to accept as being accurately known). Of course, many response problems of interest to the Navy can be fully characterized by well-known information, including information concerning the underlying mechanisms of response, and these can be effectively solved by employing quasi-deterministic solution methods. Many currently unsolved problems, however, such as the hull rupture prediction problem, involve information that is "soft," uncertain, or even lacking.

The presence of uncertainties complicates problems of response prediction and the related problems of decision making. It is helpful to separate the uncertainties into several distinct categories. First, many important variables that may be controlled and known in actual experimental tests and used to model the response may need to be regarded as uncertain in particular scenario analyses of interest. For example, in a given scenario it may be uncertain where in the vicinity of a submarine a tail-homing enemy torpedo is going to explode or, in an offensive context, where or at what depth a target submarine will be located. Second, there are uncertainties associated with missing variables. There are at least 10^{24} variables (larger, perhaps, than Avogadro's number) required for the most complete description of a given structure and only 10 to 10^2 of these are used in a typical analysis. As Murray pointed out, damage may depend on many details which the predictor cannot possibly know, even in a carefully controlled test. Many of these missing details may significantly affect the responses as well as the decisions that must be made. Third, there may be uncertainties in our understanding of the underlying physical mechanisms of response and, more generally, in the relationships between systematic response effects and the modeling variables. The engineering models required may be both complicated and undeveloped. Often, the complications will seem to increase upon closer examination of the physical phenomena as more variables are brought into consideration. To these three types of uncertainty we might add a fourth: we presently lack an understanding of how to effectively characterize and approach prediction and decision problems in the face of such uncertainties and how to put the various pieces of information at hand together in an objective and effective manner. The SDM Project has attempted to address all four areas of these concerns.

To develop the discussion, we shall refer to the different categories of information that must be considered in a rigorous analysis by special names and, in some cases, with special symbols. The set of known or hard information will be symbolized by H. Thus, H represents categorical and real variables with known values. In most cases these variables will figure prominently in models for the response. The first type of uncertainty discussed above will be termed "modeling variable uncertainty." Those modeling variables that are regarded as uncertain in the application scenario, but known in the tests that form the empirical basis of an analysis, will be denoted as U_M. Hence, the complete set of variables used to model the response is created by combining sets H and U_M. The U_M variables will sometimes be treated as real variables and sometimes as random variables; these distinctions will be apparent from the contexts of their use.

The second type of uncertainty associated with missing, ignored, or even unknown variables will be called "intrinsic uncertainty." The set of associated variables will be symbolized by U_I and always be regarded as random. The third type of uncertainty caused by a lack of understanding of the true model underlying the response will be called "model uncertainty" or "modeling uncertainty." While modeling variable uncertainties and intrinsic uncertainties are associated with variables, model uncertainty reflects our uncertain knowledge of how measurable response characteristics or effects depend functionally upon U_M and H. It is important to note that many variables that are part of the set U_I can be moved to the set U_M, and vice versa, at the discretion of the investigator who decides the scope of his or her investigation and needs. More will be said about this point later.

Figure 1 pictorially represents many of the sources of uncertainty present in the ASW warhead lethality problem. The figure is intended to illustrate the breadth of details that may, to one degree or another, be either known or uncertain. Each link in the illustrated chain is associated with informational and theoretical details, both hard and soft in character, that are relevant to the development of a solution. Categorical variables, such as the submarine design class, the types of materials, the type of warhead explosive, and the type of hull welding processes used, may be known quantities and, if so, would belong to H and otherwise to U_M. Known system design details, such as nominal structural geometries, nominal pressure hull yield stress, and charge size, may also be regarded as known and belong to H. The set of uncertain modeling variables U_M might include the torpedo warhead "hit point," the submarine depth, speed, and orientation, and structural geometric and material properties that are required for the response analysis but which are incompletely known from intelligence information. The set of intrinsically uncertain variables U_I might include details of the warhead explosive charges that vary from one charge to another, in a manner typical of the warhead design, and cause variations in output pressures. The U_I set might include details of the structural materials for a given design that would cause variations of fracture strengths and departures from nominal values of the material properties. Finally, the poorly understood and complex physics associated with fluid-structure interactions, the interactions of global deformation and fracture processes, and the causal connection between local conditions and local damage are examples that would fall into the category of modeling uncertainty.

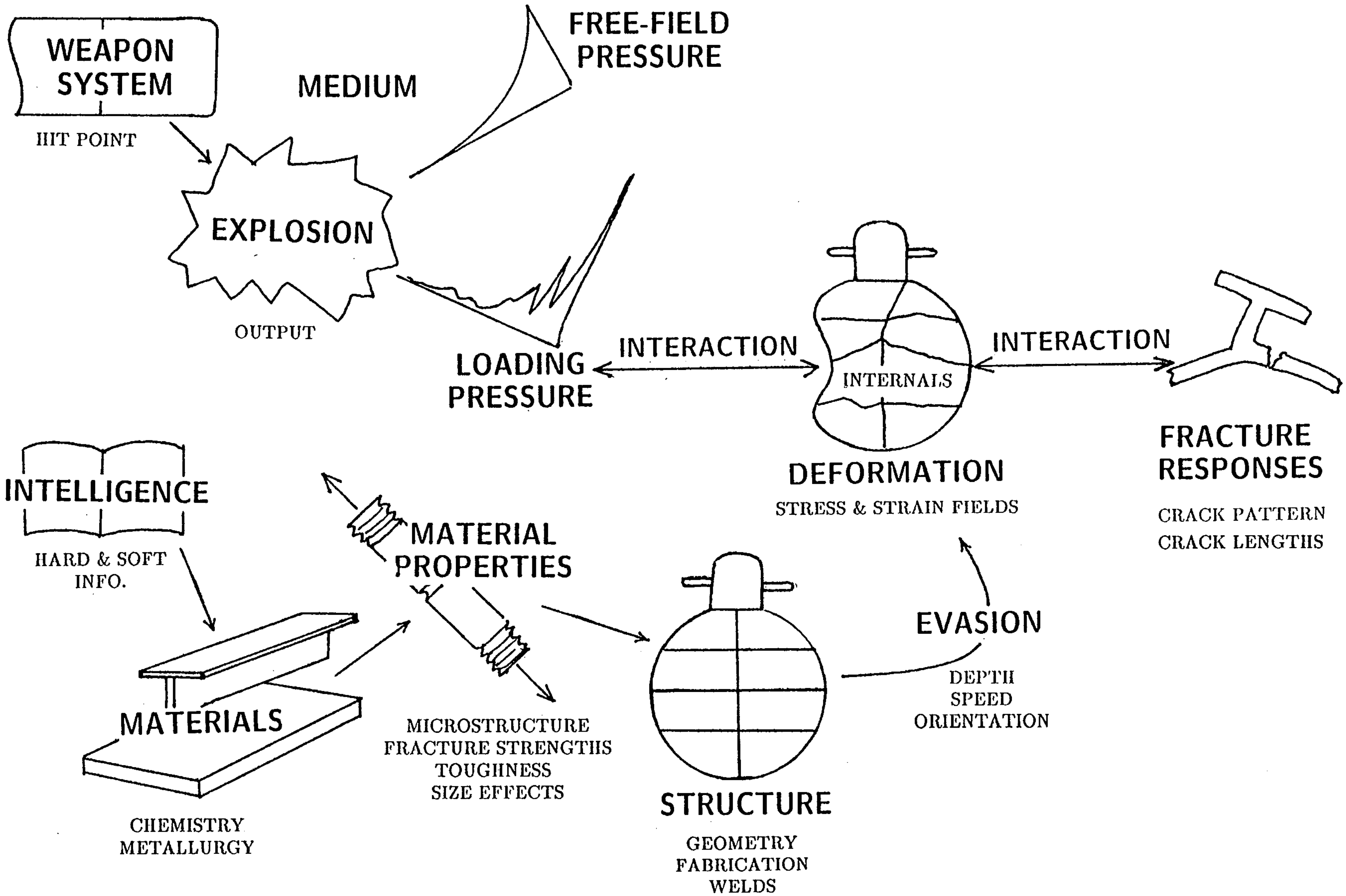

FIGURE 1. SOURCES OF UNCERTAINTY IN ASW LETHALITY PROBLEM

Formulation of Solutions:

Under the statistical modeling approach deterministic responses and effects are viewed as a subset of a broader class of stochastic or random responses and effects. Accordingly, if a response or response event is deterministic, the associated variance of the response will by definition be negligible (zero) and the probability of the event will be either zero or one. Generally, we will view responses as being nondeterministic and expect discrete event probabilities to take on intermediate values between zero and one in at least some regions of the problem domain associated with the scenario of interest. A major part of the system evaluation process, then, is the estimation of the probability of system performance, also known as the system reliability, for the conditions associated with the particular context of interest.

To focus the discussion, it is useful to consider a concise mathematical representation of the performance probability written in terms of the information available. Although this introduces a mathematical equation into this "clear language" introduction, the equation is fully explained and the narrative not difficult to follow. The equation is so central to many of the issues involved that its use seems unavoidable and necessary for a clear and direct presentation. As the reader will see, it is possible to describe not only the statistical approach, but also other approaches to the problem in the terms of the equation. An understanding of the statistical approach also requires the use of several statistical concepts such as populations, conditional means, and so on. These will be introduced as the need arises and accompanied by clarifying examples.

System performance can usually be described by reference to a capability viewed as a discrete event. For the explosion damage problem this may be some specific level of damage such as the initiation of a hull crack or deformation of the hull to a degree equal to or greater than some specified amount. Using the letter D to represent a clear definition of the desired performance, we can write the probability of damage D given the set of basic known information H as

$$P(D|H) = \int_{-\infty}^{+\infty} P(D|H,U_M)\, f(U_M|H)\, dU_M \ . \qquad (1)$$

Equation 1 tends to be known by structural reliability engineers as the "law of total probability" and by statisticians as the "law of the extension of conversation," or simply "extending the conversation." The right hand side involves $P(D|H,U_M)$, which is the

probability of D occurring when both H and U_M are controlled and known, and $f(U_M|H)\ dU_M$, which can be thought of as the (differential) probability of having the specific conditions U_M realized when conditions H hold. More rigorously, $f(U_M|H)dU_M$ is the probability of the random vector U_M taking a value in the region $[U_M, U_M + dU_M)$. We refer to $f(U_M|H)$ as the joint conditional frequency function for the U_M random variables given H. The notation and derivation of Equation 1 are discussed in the footnote.*

A hypothetical example will serve to make the use of Equation 1 more clear. Suppose that we wish to predict the probability of hull crack initiation by a specific warhead placed at a specific location and that the only uncertain modeling variable is the thickness of the hull plating. Hence, U_M represents the thickness and H represents all other modeling information. Using our intelligence information for this adversary target, we represent our uncertainty of the actual value of U_M by the density function $f(U_M|H)$. We might decide, for example, that the most likely value is 4 centimeters, which therefore locates the peak of $f(U_M|H)$, and that the function falls off in both directions to essentially zero at say 3.0 and 5.0 centimeters. We then conduct tests

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* The notation $P(D|H)$ is read as the probability that D occurs given that H is known to have occurred or be true. The vertical bar ($|$) makes the probability conditional on the occurrence of the proposition to its right and is usually read as "given" or "given that." Similarly, when a random variable A takes on discrete values only, the frequency (or generalized density) function $f(\mathrm{a}|\mathrm{b})$ means $P(A{=}a|B{=}b)$; then the integral is regarded as a sum. Here B is a random variable (or vector) of either discrete or continuous (or mixed) type. When A is a continuous random variable, $f(\mathrm{a}|\mathrm{b})$ is defined by a limiting process; then $f(\mathrm{a}|\mathrm{b})da$ means $P(\mathrm{a} \leq A \leq a + da|B = b)$. When A and a are vectors, P and f are referred to as the joint probability and joint density, respectively. In Equation 1 the notation is less explicit but customary; e.g., U_M is the value taken on by the associated random variable.

Detailed understanding of Equation 1 requires the understanding of both the "or" and "multiplication rules" of probability. The or rule states that, the probability that one or the other of two mutually exclusive events occurring is the sum of their individual probabilities of occurring. The multiplication rule says that the probability of two nonexclusive events, say A and B, occurring simultaneously is given by the product $P(A|B)P(B)$, or alternatively $P(B|A)P(A)$. Thus, from the multiplication rule the product $P(D|H,U_M) \times f(U_M|H)dU_M$ is also the differential probability that realizations D and U_M will occur simultaneously under conditions H. This could be symbolized as $f(D,U_M|H)dU_M$. Then the integral is the limit of an infinite sum of these terms, each one of which corresponds to a different value of U_M within the integral domain (note that $-\infty$ to $+\infty$ represents the entire multidimensional space, but $f(U_M/H)$ may only be nonzero over some subdomain). We note also that the "and rule" of probability states that the probability of two stochastically independent events occurring simultaneously is given by the product of their individual probabilities. Since stochastic independence of two events A and B may be defined by the condition $P(A|B) = P(A)$, or alternatively $P(B|A) = P(B)$, the multiplication rule becomes the "and" rule when events A and B are stochastically independent. While the conditioning on H on the right-hand side is necessary for a rigorous statement of Equation 1, the conditioning on H (or any part of H) may be omitted if the terms are unaffected by (or are independent of) that information.

using the specific loading conditions of interest against a series of structures of design H with known thicknesses over the range from 3.0 to 5.0 centimeters. From the resulting test data we statistically determine (estimate) $P(D|H,U_M)$, which is a function of the hull thickness U_M. Finally then, we integrate the product of the two functions in Equation 1 to estimate the desired $P(D|H)$, which is a measure of the performance of our weapon system against this uncertain target.

In this simple example we obtained the function $f(U_M|H)$ somewhat subjectively, although these judgments could have been based upon satellite photography data and optical resolution considerations. As presented, the example illustrates one of the many natural ways that relevant subjective information can be included in a statistical analysis.* On the other hand, we obtained the $P(D|H,U_M)$ function by statistical analysis of empirical data. While we might also have employed subjective information in the statistical analysis, the $P(D|H,U_M)$ function represents the principal vehicle by which the results of damage experiments are included. We will show shortly that $P(D|H,U_M)$ also represents the principal avenue through which models of the underlying physics of the structural response and failure processes are introduced.

Subpopulations and Subpopulation Characteristics:

A popular textbook on statistics begins, "The science of Statistics deals with the properties of populations."[2] Here the word *population* has been used generically. In fact, as illustrated by the above example, we are almost always concerned with subpopulations, i.e., subdivisions of a larger population that are each characterized by

* The reader may wish to inquire further into the representation and use of subjective information in statistics. The classical (frequentist) approach taken in this paper can be used to model such subjective information to a sufficient degree for important applications. However, under the Bayesian (subjectivist) approach to statistics, all knowledge is regarded as subjective. Bayesian statistics affords a rich supply of concepts and ideas that could and should be applied to the explosion damage problem in the future, particularly when highly subjective aspects dominate the problem or when the notion of probability defined as the limit achieved after an infinite sequence of tests seems awkward. This paper is presented from the classical viewpoint because this was the way the theory was originally developed. It is also thought that it is the view familiar to most readers. For further information on Bayesian statistics, the reader is referred to a paper by Singpurwalla,[3] not only for its fine summary of the Bayesian viewpoint, but also for its expository presentation of many useful statistical concepts applicable to the study of system reliability.

particular values of conditioning information considered to be known. The quantities with which we are concerned in Equation 1 are rigorously defined by testing operations conducted within these various subpopulations. In our notation, the values of modeling variables appearing in function arguments to the right of vertical lines (|) condition and identify particular subpopulations. Constant values of the conditioning information represent conditions held constant (i.e., controlled) in a subpopulation of tests that might be performed. Generally, there will be an infinite number of such subpopulations, but only a finite number will be represented in any given database. It will be instructional to consider the various subpopulations associated with the terms of Equation 1.

Consider, first, the subpopulations associated with the quantity $P(D|H,U_M)$ for which there is the most conditioning information. Although H may be held fixed, $P(D|H,U_M)$ is viewed as a function of both U_M and H. There is a subpopulation of experiments associated with each value of the variables H and U_M of interest. It is useful to think of the values of U_M and H as "labeling" each subpopulation. In the simple example considered above, one or more tests were performed for each of several, say five or six, different values of the plating thickness ranging between 3.0 and 5.0 centimeters. Consequently, in that problem we were concerned with five or six different subpopulations, each one labeled by a different U_M, H combination.

In each subpopulation of experiments in which H and U_M are fixed, the only variables remaining uncontrolled and random are the intrinsically uncertain variables U_I. These vary in some manner from individual test to individual test. Even though it is a common practice to refer to experiments conducted within the H- and U_M-labeled subpopulation as being "identical," it is only with respect to H and U_M that they may be so described, for the variables U_I are different from one experiment to another. Usually the distribution of U_I variables within the subpopulation is unknown, but assumed to be fixed.* Each actual experiment may be thought of as being a random draw from the subpopulation of experiments labeled by the specific conditions, H and U_M, by which the experiments are fully described.

* The assumption of a fixed underlying joint distribution for U_I is regarded as part of the model hypothesis as explained below. The assumption is that the tests of the subpopulation are, in a sense, statistically stable, i.e., that probabilities are well defined. Under a sophisticated experimental design some of the marginal distributions governing U_I variables may be known and the experiments randomized.

The origin and explanation of the variation of U_I is in most cases found in the actual processes used to conduct the tests and manufacture the test systems associated with each test subpopulation. The design information H and U_M usually include specific design tolerances for the test structures and testing systems that establish the bounds between which the deviations from nominal values, included among the U_I variables, are permitted to vary. The actual processes of material preparations, structure fabrications, and testing invest each subpopulation with its characteristic U_I distribution.

The quantity $P(D|H,U_M)$ is regarded as a *characteristic* of the subpopulation labeled by H and U_M. Population characteristics are quantities like means, standard deviations, variances, moments, cumulants, and other quantities that describe some constant feature of the distribution of the random variables within the population. In fact, $P(D|H,U_M)$ is a type of population mean. To see this we let the outcome of a test labeled by H and U_M be represented by a binary random variable that takes the value 1 when damage D occurs (D is an explicit definition of recognizable damage) and the value 0 when D does not occur. Then, $P(D|H,U_M)$ is the population mean of that binary random variable; i.e., it is the total number of occurrences of damage (the sum of the binary variables) in the subpopulation divided by the total number of tests in the subpopulation. If the subpopulation is infinite, $P(D|H,U_M)$ is defined as the limit that is approached by this ratio when the number of tests is increased.

We think of $P(D|H,U_M)$ as the underlying probability of damage associated with conditions H and U_M. If the damage responses under these conditions are deterministic, then the underlying probability is either 0 or 1. However, if the response of the system is sensitive to the missing information U_I, then the values of $P(D|H,U_M)$ will lie between 0 and 1. In either case, we can obtain a statistical estimate of $P(D|H,U_M)$ by summing up the number of times D occurs and dividing it by the total number of tests. Such an estimate is called a sample mean. The simple ratio of the number of failures divided by the number of tests is also called a basic estimator of the mean. It is a random function with a binomial distribution. Presumably, according to the law of large numbers, basic estimates will converge to the population mean as the number of tests increases.

The functions in Equation 1 that remain to be discussed, $P(D|H)$ and $f(U_M|H)$, are defined on a subpopulation labeled by H. Since tests in this subpopulation are only conditioned on H, the modeling variables U_M are now also random in addition to the

intrinsically uncertain variables U_I. In Equation 1 it is assumed that we have information that describes how U_M varies in the H-labeled subpopulation. This is represented by the frequency function $f(U_M|H)$. $P(D|H)$ is defined on the H-labeled subpopulation in exactly the same manner that $P(D|H,U_M)$ was defined on the subpopulation labeled by U_M and H. In fact, we could conduct tests in which the U_M variables are randomized according to $f(U_M|H)$ and obtain $P(D|H)$ in the same manner that $P(D|H,U_M)$ was obtained above. Or we could obtain it from Equation 1.

One advantage of using Equation 1 to obtain $P(D|H)$ is that we can vary scenario descriptions in hypothetical ways by redefining $f(U_M|H)$. $P(D|H)$ is then easily calculated without the need for additional testing. In the example above, $f(U_M|H)$ represented our uncertainty, due to incomplete intelligence information, in the nominal pressure hull thickness. A change in the accuracy of the intelligence information, perhaps as a result of improved collection methods, is then easily accommodated. For the lethality problem, where the target description is acquired through intelligence sources, the efficiency afforded by Equation 1 is considerable. For the survivability problem a similar efficiency is obtained when, for example, weapon and warhead characteristics are obtained from intelligence sources.

Comments and Summary:

We have not yet addressed the topic of modeling uncertainty because such a discussion has not yet been needed. It will be discussed in detail later in the section that describes the statistical approach. From the statistical viewpoint, modeling is used to connect together (to pool) various probability estimators, like the basic estimators discussed above (which are usually inaccurate due to sparseness of the data), to obtain more accurate estimators of the population characteristics. In the above discussion we saw that we could, in principle, obtain accurate estimates of these characteristics simply by conducting enough tests under the right conditions; no physical modeling or combining of data collected under different test conditions was required. It was assumed that the physics was correctly represented in the tests. Because testing is usually expensive, however, the idea of estimating the probabilities of interest by empirical means alone is usually impractical and inefficient. The pooling together of test information collected under varied conditions to obtain new estimators that are more accurate than the simple basic estimators is essential, then, for an efficient

approach. Physics and engineering models for the systematic effects may be used to make the necessary connections among the empirical data associated with the various test conditions. Under the statistical approach the adequacy of these models becomes a statistically testable condition or hypothesis. Thus, modeling uncertainty will be handled below in a different manner from the way in which we have treated the uncertainties in the variables.

In summary, the casting of the explosion damage problem in probabilistic terms allows one to address issues involving uncertainty. Uncertainties, which are always present, (1) may affect the responses of the tests, (2) are often involved in the description of various application scenarios of interest, and (3) always affect the processes of decision making. Equation 1 describes how uncertainties contribute to the probability of system performance; it may be applied in both system lethality and survivability contexts. The use of Equation 1 as well as other probabilistic models useful for making decisions requires that the problem be characterized and formulated in probabilistic terms. Because random effects are fully modeled under the statistical approach in addition to the mean systematic effects, the statistical approach is more detailed and rigorous than the alternative semi-empirical and quasi-deterministic approaches which model some, but not all, of the systematic aspects of the responses.

SEMI-EMPIRICAL APPROACH

The semi-empirical methods that have been developed by the Navy and applied to explosion damage prediction problems have many features in common with the statistical approach and with the quasi-deterministic approach, both of which are described below. The semi-empirical approaches rely heavily on a rudimentary form of statistical analysis often described as curve fitting. The response variable to which functions are fitted, perhaps using least squares regression, is usually chosen to be a continuous variable, such as a measure of the inward deflection of the pressure hull. This is done even when the response of primary interest, for example hull rupture, is of a discrete nature. The continuous response variable is then fitted to functions of independent variables describing the structure and test conditions. The fitting process is typically accomplished by assigning values to various coefficients present in the fitting functions to produce agreement, in some sense, with the data. The forms of the fitting functions are often suggested by data plots.

In some models the independent variables used are generalized coordinates that are functions of the variables describing the structure and test conditions. The forms of these functions are often suggested by dimensional analysis considerations or by simplified deterministic response models. Examples of functions used as independent variables, taken from the submarine damage prediction literature, include a normalized hoop collapse stress formula and the ratio of bending moment to plastic limit moment for a stiffening ring.

The strategy for predicting damage that is usually employed under the semi-empirical approach is to first obtain a formula for the selected continuous response variable, and then to predict the damage of interest by assigning a critical limit or range of critical limits to the response variable that might be empirically associated with one or several degrees of that damage. The assignment of such critical values is usually done by making personal judgments based upon knowledge of the empirical results. Prediction of damage is then accomplished by comparing a calculated response value, obtained by inserting new conditions of interest into the fitted formula, with the critical values. The combination of the response variable and its critical value is sometimes referred to as a failure criterion.

We will show below that the statistical approach is an extension and rigorous refinement of the semi-empirical approach. The critical values that play an important part in the expression of the failure criteria under the semi-empirical approach are values that are intended to be associated with high, low, and sometimes mid-range values of failure probability. Quantities like $P(D|H,U_M)$ and $P(D|H)$ thus appear tacitly and qualitatively. Instead of using statistical methods for formally establishing the links between critical values and probabilities, the associations are usually made under the semi-empirical approach by invoking judgment. In some system response applications $P(D|H)$ has been calculated by means of Equation 1 (or through an equivalent expression to be introduced below), but in these applications $P(D|H,U_M)$ has been modeled as a deterministic step function (0 or 1) to simplify the analysis.

Semi-empirical prediction techniques work reasonably well when the test conditions of interest are not too far removed from the conditions upon which the prediction rule has been based, when rough estimates of risk are sufficient, and when the consequences of errors are small. When interpolations and extrapolations are required that are remote from the data conditions or when one needs more quantitative assessments of risk, the limitations of the semi-empirical approach become more apparent. The basic criticism of the approach is simply that it lacks rigor and, consequently, that it is often unable to meet more demanding needs.

QUASI-DETERMINISTIC APPROACH

The basic differences between the statistical approach and the quasi-deterministic approach can be rather clearly stated in the terms of the above discussion. The quasi-deterministic approach is an example of extensive reductionism, i.e., the breaking down of a complex process into smaller parts, each of a more elementary nature. In the author's opinion the approach may be fairly characterized as an attempt to make the set of uncertain modeling variables U_M as large as necessary for it to be possible to model the response D in a nearly deterministic, or quasi-deterministic manner. Equivalently, it is an attempt to make the set of intrinsically uncertain variables U_I small enough that their effects upon $P(D|H,U_M)$ might be ignored. Under these circumstances D can be predicted with certainty and $P(D|H,U_M)$ can be expected to take on the value zero or one within known regions of the H,U_M space of modeling variables.

Under most quasi-deterministic approaches the macroscopic definition of damage we have symbolized as D would be modeled as the culmination of more elementary damage processes. These basic damage states are generally effects observed in laboratory tests of small, highly prepared material specimens such as unnotched, notched, or fatigue-precracked round tensile bars, bend bars, and compact tension (CT) specimens. Typically, the specimens are of designs rigorously prescribed by the American Society for Testing and Materials (ASTM) or some other standardizing institution. The basic testing goal is to tightly control all specimen variables except those that randomly vary within the material microstructure in some fashion characteristic of the material type. Hence, the presence of U_I variables within the material microstructure is acknowledged.

The small specimen test results are used to characterize the behavior of material elements up to the point of failure and also after failure has occurred. Relationships describing the local behavior prior to the time of failure are normally referred to as constitutive models. Relationships used to describe the behavior of failed material elements are sometimes called failure models or damage models, although the difference between constitutive and failure models is not necessarily distinct. Both types of

models express relationships among the dependent state variables that are in turn functions of the modeling variables. They may be viewed as constraint systems that are used to close and render solvable the system of continuum-mechanics-based equations that govern the behavior of the material elements when the elements act collectively, i.e., as parts of the continuum.

Constitutive models, for example, might describe specific types of material responses such as elastic, plastic, viscoplastic, or thermo-elastic-plastic behaviors of isotropic or orthotropic materials. Generally, constitutive models concern processes of shear and dilation of the material elements that occur through dislocation and thermal expansion mechanisms. Damage models, on the other hand, are attempts to account for more discontinuous processes that might lead to the development of fracture surfaces. For ductile materials damage processes of primary concern are the nucleation, growth, and coalescence of void spaces within the material. For most materials damage models are currently less well developed than constitutive models. Fractures also introduce new boundary conditions that can be problematic. Scatter is present in all small specimen data though to a lesser degree for quantities modeled by the constitutive equations. Both constitutive and failure models are based upon average properties of observed responses and measurements. For example, an idealized damage model might be based upon a cubic lattice of void nucleation sites. An experimentally determined mean distance between void nucleation sites might then be used as the lattice spacing in the model. Magnitudes of scatter associated with elementary failure effects typically run at five or ten percent of mean values for the ductile steels used to fabricate modern submarine hulls.

Usually, the occurrence of a particular macroscopic damage condition D can be easily recognized by running the quasi-deterministic code for particular controlled values of H and U_M. Then, because the effects of U_I variables are assumed to be negligible, the probability function $P(D|H,U_M)$ is assumed to take on the values 0 and 1 over various regions of the H,U_M space as directed by the failure model. The quasi-deterministic response codes can be viewed as schemes for partitioning the space of modeling variables into such dichotomous probability regions. Of course, these regions will be in agreement with the empirical data (i.e., damage D will be observed only in regions that have been assigned probabilities of one) only if the models underlying the codes correctly represent the behaviors of the structures and testing systems, and if the effects of the U_I variables on the response D are truly negligible.

Under the quasi-deterministic approach, uncertainties that are absent from the small specimen tests but present in problems involving complex structures such as submarines are expressed by the function $f(U_M|H)$. The effects of these uncertainties upon $P(D|H)$ are then calculated using Equation 1 and the deterministic $P(D|H,U_M)$ function. This is perhaps usually accomplished by substituting an alternative form of Equation 1 expressed in terms of the distribution of some state variable function of H and U_M (transformations of this kind are discussed in detail on p. 35). Such analyses represent proper investigations of the effects of modeling variable uncertainties on the response of interest. Sometimes $f(U_M|H)$ is less appropriately employed (in the author's opinion) to express the effects of uncertainties of other types. Attempts are sometimes made to express the effects of intrinsic uncertainties, which were ignored in $P(D|H,U_M)$, within $f(U_M|H)$ (or, equivalently, within the mathematically related state variable distribution). For example, the scatter in the small specimen data, used to derive the constitutive and failure models, is sometimes accounted for in this way by adding it to the effects of the U_M variables upon the state variable distribution (usually the standard deviations or variances are added). This add-on line of thinking is sometimes extended to also provide a hedge against modeling uncertainty, i.e., the uncertainty in the modeler's knowledge of the underlying physics (perhaps thought to be responsible for the disagreement between the model theory and the data). As we shall see, the statistical approach provides a more natural and less ad hoc way of handling the intrinsic and model uncertainties.

Under the quasi-deterministic approach, variables are included within the U_M set that would be included as part of the U_I set under the semi-empirical and statistical approaches. Because the U_M set is large, the task of specifying the joint modeling variable distribution $f(U_M|H)$ for the class of structures and scenarios of interest may be quite difficult and quite expensive. Hence, the quasi-deterministic approach places a significantly greater burden on the problem of specifying the joint density function $f(U_M|H)$ than do the alternative approaches. For any reliability analysis to be accurate, one must be careful to put back the randomness and uncertainties that were essentially removed from consideration during the process of determining $P(D|H,U_M)$.

There is, as an appropriate part of the quasi-deterministic approach, an extensive literature on statistical distributions associated with details of micromechanical responses and on the distributions governing fluctuations of various aspects of loadings associated with winds, waves, and other natural forces. While such studies are valid

and useful, there is often in practice a tendency to ignore other important sources of uncertainty and to view response uncertainties solely in terms of material and loading issues. Other sources of uncertainty in military problems of interest that may be consequential include geometric variations from nominal or design geometries, geometrical and material irregularities associated with weldments, poorly understood and represented joints of all types, material heterogeneities, residual strains, and loading fluctuations associated with variations in warhead performances. For the lethality problem the most basic of the design variables, such as the nominal pressure hull diameter and thickness for example, may be highly uncertain. In many cases, the distributional properties of the uncertain variables will have to be estimated empirically by examining structures like those for which predictions are sought. In some cases, the distributional properties will have to be assigned by a priori means based upon expert opinion.

As alluded to above, model uncertainty is an issue of intense concern to practitioners of the quasi-deterministic approach due to the extensive scope of the modeling attempted. Because of the complexities of the underlying physics, many aspects of the models used in structural response codes are known to be approximate. Important questions, then, concern how these approximations affect or limit the predictive power of the code. Resolution of the issue requires comparisons with experimental results. Currently, code performance is typically assessed by making comparisons with the results of controlled benchmark tests not unlike the small laboratory tests used to calibrate the material models. The agreement between theory and data, however, is almost never perfect. The issue of model validation becomes more complicated when comparisons are made with the results of large structure tests due to the additional sources of error that might then be acting. Regardless of the test size, model uncertainty is a statistical issue that requires a detailed accounting for all significant sources of uncertainty. The statistical approach for dealing with this issue found in the next section is consistent with a long-standing body of statistical theory.

While the quasi-deterministic approach has been highly successful in applications to structural design, it has been less successful when used for predicting structural failure. This is because engineered systems are usually designed to operate rather than fail. They are designed to operate within specified limits, in a deterministic manner, according to well-understood physical principles. Barring significant design errors, this enables deterministic engineering theories to predict the systematic behaviors of

engineered systems across a wide range of conditions described by the design variables, H and U_M. Usually, failures occur when systems are taken beyond the bounds of the design domains (by system overloads and erosion processes), which renders the system behaviors more sensitive to details contained in U_I.

The quasi-deterministic approach may be criticized for effectively maintaining that it is possible to control enough variables for problems to be made deterministic. For some responses and some materials the approach is a reasonable strategy for achieving a reasonable level of accuracy. But, the difficulties encountered when the quasi-deterministic approach is used to predict the responses of brittle materials suggest that the approach is lacking in ways that are of fundamental significance. The absence of consideration of the random effects places issues such as validation and accuracy outside the scope of the quasi-deterministic approach. Furthermore, the gains made by controlling variables are offset by the increased costs of obtaining accurate estimates of $f(U_M|H)$.

STATISTICAL APPROACH

The goal of the statistical approach is to develop a probability model for the responses of interest that is sufficiently valid throughout the modeling variable domain of interest so that the model can be used for making realistic predictions with quantifiable accuracies and, more generally, for making objective decisions.

If submarines and testing were cheap, we would solve the hull rupture prediction problem by simply building and testing submarines according to the information H and then estimating $P(D|H)$ to any desired level of accuracy using the ratio of successes to total number of tests. In contrast, if all we have are the tests of small specimens of the materials of interest, then we need the models of the quasi-deterministic approach in which the larger set of modeling variables necessary for obtaining a solution from the small specimen data is controlled. Thus, the nature of the available data drives the necessary analysis; the models required may be regarded as models for the databases. As we have seen, however, the quasi-deterministic solution is not complete until the uncertainties that are removed for the determination of $P(D|H,U_M)$ are put back into the problem. This requires the specification of $f(U_M|H)$ which may be expensive when large numbers of variables are controlled.

The statistical approach is a very flexible approach for modeling response data. Because of its flexibility, the approach offers a way of making use of more of the available data than the two alternative approaches we have discussed. Consequently, it offers the possibility of obtaining more optimal solutions to complex structural response problems. In principle the statistical approach can be applied to all types of response data: to small laboratory specimen tests, to tests against full-size prototypes, and to data that fall in between.

The statistical approach was developed because of a desire to make full use of large- and intermediate-scale test data obtained by the Navy over several decades. In particular, we wished to model a database that includes a wide variety of reduced-scale models of submarine pressure hulls. Some of the pressure hull models were fabricated by rolling and welding together plates; other models, including the stiffeners, were

machined out of thick-walled pipes. These models were then explosively tested throughout a wide range of loading conditions to simulate hull failure responses. Although structure designs were sometimes repeated in the database, there were few, if any, strict test replications. While much of the testing was performed for the purpose of developing the semi-empirical prediction models, the database provides a wealth of information that has never been fully utilized.

We begin our detailed discussion of the statistical approach with some simple examples designed to illustrate the application of statistical modeling techniques to the hull rupture prediction problem. It will be readily apparent that the complexity of the required analysis increases with the number of modeling variables that are varied in a controlled manner within the test population.

Example 1 -- Single Discrete Response; All Modeling Variables Constant in Tests:

Each test of a submarine pressure hull model results in a continuum of responses that occur across and throughout the structure, develop in time, and interact in complex ways to produce a final pattern of damage. Depending on the severity of the loading, the damage pattern may include deep dishing of the hull plating, severe distortions and fracturing of the stiffening rings, and possibly one or more tears in the pressure hull material. If we define D to represent some condition of interest with respect to the final pattern of responses, such as at least one tear in the pressure hull, we can ask what is the probability of D occurring under the known conditions of the test. As we have seen, for a particular set of test conditions there is a basic estimator that converges to this probability as the number of test replications is increased. The analysis required to estimate the probability in this manner, simply by performing replications of tests, is trivial, regardless of the detailed complexity of the responses, if we are interested in only one fixed set of problem conditions. Some elements of the required statistical analysis are illustrated in Figure 2.

Example 2 -- Single Discrete Response, One Modeling Variable Varied in Tests:

In an earlier example we used Equation 1 to predict the probability of hull crack initiation when the only uncertain modeling variable was the thickness of the hull

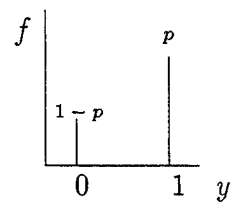
Response of interest: Damage event D (a specific definition).

Response variable: Bernoulli (binary) random variable Y_j, where

$$Y_j = \begin{cases} 1 & \text{if } D \text{ occurs in } j\text{th trial,} \\ 0 & \text{if } D \text{ does not occur in } j\text{th trial.} \end{cases}$$

Response probability model: Bernoulli frequency function

$$f_{Y_j}(y;p) \;=\; p^y\,(1-p)^{1-y} \;,\; y = 0,\,1;\;\; 0 \le p \le 1 \;.$$



Database: Vector of results from n independent identical Bernoulli trials

$$\underline{y} \;=\; (y_1,\, y_2,\, \ldots\,,\, y_n)'$$

Database probability model:

$$f_{\underline{Y}}(\underline{y};p) \;=\; \prod_{j=1}^{n} f_{Y_j}(y_j;p) = p^{\Sigma y_j}\,(1-p)^{n-\Sigma y_j} \;,\; y_j = 0,\,1,\; j = 1,\,2,\,\ldots\,,\,n\;.$$

(Becomes a binomial frequency function upon transformation of variables.)
Here p is an unknown parameter, $0 \le p \le 1$.

Parameter (basic) estimator: $\hat{p} \;=\; \sum_{j=1}^{n} Y_j \Big/ n\;.$

Basic estimator mean and variance:

$$E(\hat{p}) \;=\; p, \qquad V(\hat{p}) \;=\; p\,(1-p)\big/ n\;.$$

FIGURE 2. ELEMENTS OF A STATISTICAL ANALYSIS WHEN ALL MODELING VARIABLES ARE CONSTANT

plating. In that problem there was only one U_M variable. Although somewhat more involved than the trivial case above, a straightforward solution to the problem again seemed possible. The probability function $P(D|H,U_M)$, viewed as a function of U_M, was evaluated by building and then testing structures with five or six different values of hull thicknesses. This created five or six different basic estimators of the probability function $P(D|H,U_M)$, one associated with each of the U_M values at which tests were performed. As illustrated in Figure 3, we can easily imagine the observed values of the estimator (its estimates) falling about a curve representing the underlying mean function $P(D|H,U_M)$ [$p(x)$ in Figure 3]. The appropriate statistical model by which $P(D|H,U_M)$ can be estimated in this case is called a regression model. By definition, regression models are concerned with the estimation of conditional means. Because the response variable, in this case, is the discrete basic estimator, quantal or discrete response regression models are required. Statistical details concerning such models may be found in references 4, 5 and 6, and at a more advanced level in reference 7.

Because linear least squares regression is a related topic familiar to many readers and involves concepts that also apply to discrete response regression models, it may be instructive to review the basic ideas. The problem is usually framed in terms of a normally distributed response variable Y and a known independent (regressor) variable x. It is immaterial whether x is known before the experiment, or afterwards as the measured realization of a random variable X.* The primary things of interest in such problems are the mean and variance (or standard deviation) of the response variable Y conditional upon various values of x. Under most treatments the variance is assumed to be an unknown constant. We describe the mean function as the expected value of Y given that $X = x$, written as $E(Y|x)$. Under linear least squares regression theory a functional model for the relationship between the mean $E(Y|x)$ and the regressor variable x is selected that is linear in the model parameters.† For example, denoting some known functions of x as $h_1(x)$ and $h_2(x)$, we might hypothesize that $E(Y|x)$ has the form $E(Y|x) = \alpha + \beta h_1(x) + \gamma h_2(x)$; the right-hand side is often called the regression function. Frequently, the form of the regression function is based on a data plot of y versus x. Values of the model parameters α, β, and γ are then chosen which minimize the sum of the squared errors [i.e., the squared deviations of the observed Y values from

* As in Figures 2 and 3, we adhere to the convention of denoting random variables by upper case characters and of using lower case characters to denote the observed values or realizations.

† Often the linearity attribute is mistakenly thought to apply to the regressor variables rather than the model parameters.

Response of interest: Damage event D (a specific definition).

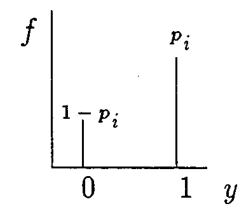
Regressor variable: An arbitrary modeling variable, say x.

Response variable: Bernoulli (binary) random variable Y_{ij}, where

$$Y_{ij} = \begin{cases} 1 & \text{if } D \text{ occurs in } j\text{th trial at level } x_i, \\ 0 & \text{if } D \text{ does not occur in } j\text{th trial at level } x_i. \end{cases}$$

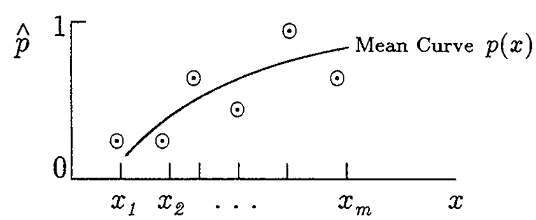
Response probability model: Bernoulli frequency function

$$f_{Y_{ij}}(y;p_i) = p_i^y (1-p_i)^{1-y}, \; y = 0, 1; \; 0 \le p_i \le 1 .$$



Database: m vectors of results from conditionally independent (defined on p. 41), identical Bernoulli trials $\underline{y} = (y_{11}, y_{12}, \ldots, y_{1n_1} | y_{21}, y_{22}, \ldots, y_{2n_2} | \ldots | y_{m1}, y_{m2}, \ldots, y_{mn_m})'$, where n_i is number of tests at ith level. Because test order is unimportant, we can equivalently express the data base in terms of binomially distributed basic estimators defined as $\hat{p}_i = \sum_{j=1}^{n_i} Y_{ij} \Big/ n_i , \quad i = 1, 2, \ldots, m$,

and report the data as binomial proportions and numbers of tests $\{\hat{p}_i, n_i\}$ at specific regressor levels. Plotted $\hat{p}_i$ values will fall about means p_i, $i = 1, 2, \ldots$, m.



Database probability model: $f_{\underline{Y}}(\underline{y};\underline{p}) = \prod_{i=1}^{m} \prod_{j=1}^{n_i} f_{Y_{ij}}(y_{ij};p_i) = \prod_{i=1}^{m} \prod_{j=1}^{n_i} f_{Y_{ij}}(y_{ij};p(x_i))$.

To reduce number of parameters, model is usually reparameterized by substituting a class of (regression) functions for $p(x)$. Estimator details depend upon chosen class.[4-7]

FIGURE 3. ELEMENTS OF A STATISTICAL ANALYSIS WHEN ONLY ONE MODELING VARIABLE IS VARIED

the mean, $(Y - E(Y|x))^2]$, where the summation is over all observed pairs of x and Y. Usually, the estimators of the parameters are obtained by inserting the regression function into the summation and setting to zero the partial derivatives with respect to the parameters. An estimator for the assumed-to-be-constant variance of Y is usually obtained in terms of the sum of the squared errors of the responses relative to the estimated mean.

The fitted regression model can be used to estimate the unknown conditional means associated with the tests. For example, by plugging in the different test conditions represented, say, by x_1, x_2, . . . , x_m one obtains the estimates $E(Y|x_1)$, $E(Y|x_2)$, . . . , $E(Y|x_m)$. The model can also be used to obtain estimates at untested values of x. These values represent interpolations and extrapolations of the model. It can be shown that regression estimators of the mean are more accurate than the individual basic estimators associated with each test. This is because the regression model permits the pooling together of responses from dissimilar tests as a result of the additional structure provided by the regression function. Increased accuracy is demonstrably true, however, only if the assumed structure for relating one conditional mean to another, represented by the regression function, is correct. Thus, after the fitting process, the hypothesis that the regression function can be used to represent the true mean structure is usually tested by a statistical goodness-of-fit test. Other tests of residuals might also be applied, e.g., to test the constant variance hypothesis. In general, a statistical hypothesis test is designed so that if the hypothesis is true, the random test statistic (calculated from the same data used to estimate the model parameters) will have a known distribution. If the observed value of the test statistic is rare, based on its frequency of appearance in the distribution, this is taken as evidence that the hypothesis being tested is questionable. Consequently, if the observed value of the test statistic falls below a chosen threshold (called the significance level), the model is rejected; otherwise, the model is tentatively accepted and used.

For small data sets the probability of model rejection under the model hypothesis test is less than for larger data sets. This is to be expected, perhaps, because there is less information about the underlying process contained in the smaller data set. However, the price paid for easy model acceptance is that the accuracies of the regression estimators derived from smaller data sets are not as good as those of successfully tested models based upon larger data sets. The accuracies are reflected in

the widths of the estimator distributions, perhaps measured in terms of confidence limits or standard deviations. Thus, the model hypothesis test forces the model into closer agreement with the actual systematic structures underlying the data as more data are acquired, otherwise the model is rejected. The payoff for the experimental and modeling efforts comes in the form of greater estimation and prediction accuracies. This convergence of the model and estimated quantities to the underlying population characteristics as more data are acquired is called by statisticians model consistency. These basic details of the modeling strategy employed under linear least squares regression theory apply, in large measure, to classical parametric statistical models of quite general forms.

Example 3 -- Single Discrete Response, Many Modeling Variables Varied in Tests:

In the Navy database of tests against stiffened shells the differences between test conditions are nearly always described by changes of more than one U_M variable. Hence, in this broader problem, we are concerned with basic estimators of $P(D|H,U_M)$ at multiple points within the H,U_M space of modeling variables. In principle, this problem could be approached as a multiple-variable regression problem. Such an approach would be similar to the description of the single-variable regression problem we have just discussed, the principal difference being that x would be regarded as a vector. Under a multiple-variable regression approach $P(D|H,U_M)$ would be regarded as an unknown mean hypersurface within the H,U_M space about which the basic estimators randomly vary. Such a surface is sketched in three dimensions in Figure 4. The main problem with using the multiple-variable regression approach for this problem lies in the difficulty of realistically expressing the form of the hypersurface, i.e., the form of the regression function. The systematic physical response of the stiffened shell varies in very complex ways throughout the H,U_M space of modeling variables as response mechanisms change and interact in complex manners. Hence, the multiple-variable approach may be of use in small neighborhoods of the space, but not over the expanses represented by the database. Approaches using multi-linear or quadratic forms of the regressor function are sometimes called response surface techniques.

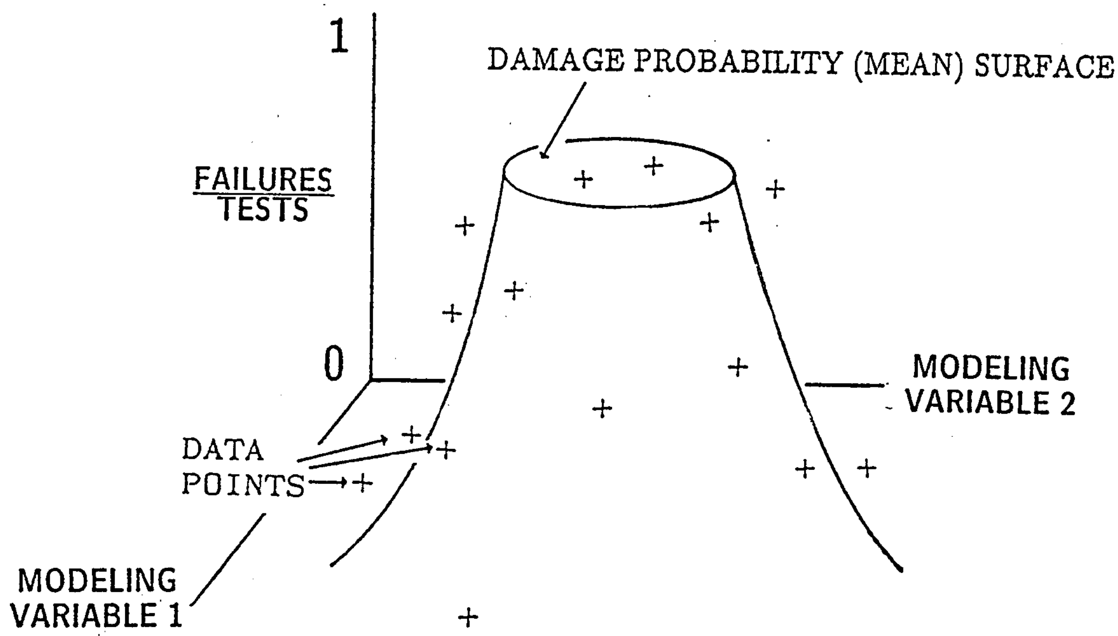


FIGURE 4. EXAMPLE OF A MEAN SURFACE IN A MULTIPLE-VARIABLE REGRESSION PROBLEM

More General Discussion of Statistical Modeling:

Before describing how engineering and physics models can be used to expedite the formulation of the probability model, it will be useful to elaborate upon some of the concepts introduced above and to introduce some additional ones. First, we will attempt to explain more clearly what is meant by the terms *nondeterministic responses* and *probability model.* This is important because the development of a probability model was cited above as the primary objective of the approach.

Earlier, we had stated that under the statistical approach all responses will be regarded as nondeterministic. This means that any response variable, say Y, can be associated with a distribution that describes the frequency in which the different possible values of Y appear in the population of interest. Deterministic responses are not excluded from this viewpoint; rather, they are associated with degenerate distributions, such as delta functions, whose probability masses are concentrated at single points. In general, we may be interested in discrete or continuous responses whose possible values might be spread across a wide range of values or concentrated at a single point.

The distribution of a response Y can be represented by a frequency function (e.g., density) or by a (cumulative) distribution function. Either and any representation of the distribution is a probability model for Y. That is the meaning of probability model. It is a (hypothesized) complete description of the way in which Y varies. It represents (hypothetically) all there is to know about the random variable Y.

The constant, mathematically derivable features of distributions are its population characteristics. As we have said, these include quantities like means, standard deviations, moments, cumulants, and so on. Most popular distributions can be fully described by two or three such terms; but more complex distributions may require many such constants for a complete description. Any characteristics in addition to the minimum number required to fully describe the distribution would then be viewed as dependent functions of the minimum set. The parameters in terms of which a distribution is usually expressed, such as the threshold, scale, and shape parameters of a Weibull distribution or the mean and standard deviation of a normal distribution, are also population characteristics that may be expressed in terms of this minimum set. In fact, the minimum set may be comprised of these parameters. Hence, a probability model can be described by specifying the functional form of the distribution and a set of population characteristics.

Generally, the "true" probability model underlying a natural real-world response is unknown. We obtain information about the model by collecting data; i.e., by making observations and measurements of the response experimentally. Statistical models are estimates of the true probability model underlying a random response. That is, statistical models take data into account so that the model describes to a quantifiable level of accuracy the behavior of the response. Usually, this is done by developing functions of the random data whose values converge to the unknown population characteristics as more and more data are included. Such functions are called consistent estimators of the population characteristics. Estimators, that are not consistent are said to be biased.

If we are concerned with a single population, the population characteristics are simple constants. But, if we wish to vary, in a controlled manner, a number of modeling variables in the tests, subpopulations are generated and the population characteristics become functions of the modeling variables. We have seen that multiple sets of unique test conditions create multiple subpopulations with which we must be concerned. It is natural then to include the notion of functions that describe

the variation of the population characteristics from one subpopulation to another. The probability model for the random response variable is then specified, for example, by means, variances, etc. that are functions of the modeling variables varied in the tests.

The population characteristics are said to represent the systematic behavior of the random response. When many subpopulations are of interest, it is common to refer to the functions describing the variation of the population characteristics as the *structure* underlying the probability model. Thus, a model for the systematic behavior of a response might involve a model for the underlying mean structure, a model for the underlying variance structure, and so on, until the probability model is fully specified.

Classical physics and engineering models of complex systems, such as submarines, can be used to develop models for the mean structures underlying the system responses. Up to the onset of failure, engineering models will usually describe the responses of an engineered structure, in an average sense, with reasonable accuracy. Engineering models can also be used to model the mean behavior of a structure after a failure process has begun when the geometries and mechanisms of the failure process can be adequately represented. Failure processes that are difficult to model deterministically are those that involve instabilities, such as buckling, and those that involve fractures and the growth of cracks. The modeling of fracture is difficult not only because the responses may be affected by the variabilities within the material microstructure, but also because the responses are sensitive to the local stress and strain fields which are difficult to predict with accuracy in a complex structure. In both cases the modeling difficulties may be attributed to intrinsic uncertainties operating within each structure and to modeling uncertainty, i.e., uncertainty in the description of the underlying physics.

Modeling a Single Continuous Response Variable:

A well-known approach for statistically modeling the response of a single continuous response variable is to add a random error term to some central measure of the response and then estimate the parameters of the error distribution. For example, suppose we wanted to predict the length of a coupon crack in a static test as a function of the maximum applied load. We might use a fracture mechanics model to predict the length of the crack under the various loading conditions of interest and obtain a model for the random responses actually observed by adding error terms to the

fracture mechanics predictions. Using test data obtained for a number of known loads, we could plot error (i.e., the difference between the predicted and observed responses) as a function of load and then estimate the mean and standard deviation of the random error term by simple linear least squares regression. In this we might make the assumptions that the errors are normally distributed with constant but unknown mean and variance. The estimated probability model for the crack length response would then be essentially the same as the model for the errors but with a mean structure equal to the sum of the fracture mechanics prediction and the estimated mean of the error term. The goodness-of-fit test of the probability model is also a test of the hypothesis that the fracture mechanics solution is proportional to the true conditional mean for each value of the load.

Modeling a Single Discrete Response Variable:

The techniques for modeling discrete random responses, i.e., responses regarded as random discrete events, are less well known. Since we were concerned with discrete events in our discussion of Equation 1, it is not surprising that Equation 1 can be used to describe the discrete response modeling techniques. To connect Equation 1 with the physical models, however, it is necessary to reexpress it in a manner frequently found in reliability analyses. We write

$$P(D|H) = \int_{-\infty}^{+\infty} P(D|H,\Upsilon)\, g(\Upsilon|H)\, d\Upsilon \,. \tag{2}$$

The continuous variable here denoted by Υ (upsilon) is usually referred to in the reliability literature as either a "load function" or a (generalized) "stress function." * Υ is a function of H and U_M. Just as U_M is regarded as random within the integral of Equation 1 [distributed as $f(U_M|H)$], so too is Υ random under the integral of Equation 2 [and distributed as $g(\Upsilon|H)$]. It is easily shown that Equation 2 is obtained from Equation 1 by a transformation of variables from the set H and U_M to the variables H and Υ. The density function $g(\Upsilon|H)$ is completely defined by its relationship to $f(U_M|H)$ under the transformation of variables.

* In some problems it is advantageous to let Υ be a vector. Equation 2 is still valid when Υ is regarded as a vector of intermediate (generalized) "stress" functions.

It is instructive to rigorously discuss the conditions under which Equation 2 can be obtained from Equation 1. These are summarized in the next several paragraphs. A more complete discussion is given in reference 4.

The transformation function $\Upsilon(H,U_M)$ may not be chosen in a completely arbitrary manner, for it is required that constant values of Υ be associated with constant values of the probability of damage D. Otherwise, we could not write $P(D|H,U_M)$ as $P(D|H,\Upsilon)$, which is what is done to get from Equation 1 to Equation 2. For this substitution to be rigorous, projections of constant probability surfaces (level surfaces) onto the space of modeling variables, as shown in Figure 5, must represent functions of the form $\Upsilon(H,U_M) = \Upsilon$, where Υ values are constant on each level surface. Furthermore, since the mean structure $P(D|H,U_M)$ is unknown, we cannot know for certain, when we choose $\Upsilon(H,U_M)$, if this condition is satisfied. Hence, we must treat our choice of this function as a hypothesis and regard it as part of the overall probability model hypothesis. In this sense, then, the form assumed for $\Upsilon(H,U_M)$ becomes a statistically testable assumption.

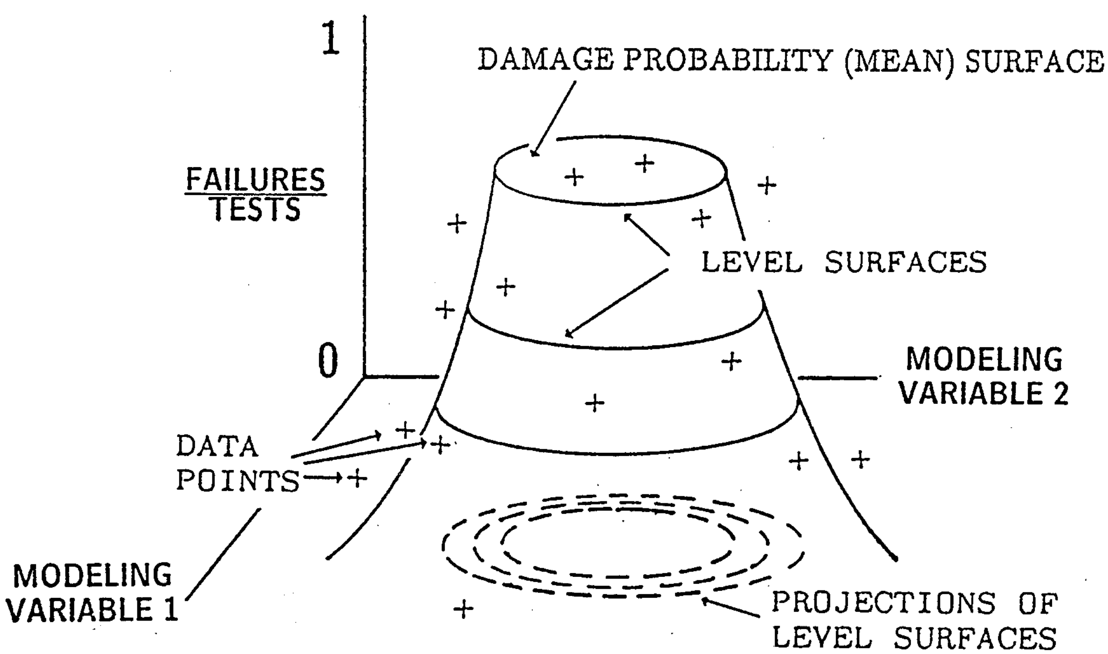


FIGURE 5. PROJECTIONS OF LEVEL SURFACES ONTO SPACE OF MODELING VARIABLES

In the usual context under which Equation 2 is applied, $P(D|H,\Upsilon)$ is regarded as a monotonically increasing function of Υ that is zero when Υ is zero. This is in accord with the usual notions of loads and stresses. In the engineering literature, however, the words *load* and *stress* have specific physical meanings which can introduce confusion when the terms are applied in a strict mathematical sense as in Equation 2. The author prefers, therefore, a more neutral and specific language. He refers to the class of functions whose values actually do label constant probability levels as damage or response potentials. Then, the higher the value of the damage potential associated with an experiment, the higher the value of the damage probability. Because the damage potential functions are unknown, the functions that are hypothesized to be damage potentials, and used in practice for modeling purposes, are distinguished from damage (or response) potentials by calling them damage (or response) indices. Thus, damage potentials are unknown and damage indices are our attempts to represent them. We use the symbol Υ to refer to both, and the distinction is understood from the context. Clearly, the process of selecting a damage index requires physical intuition regarding the unknown damage process.

When Υ is a damage potential $P(D|H,\Upsilon)$ can be represented by a probability distribution function. Let us denote this as $F(\Upsilon)$. We will call the random variable that $F(\Upsilon)$ distributes a "strength," which is the word used in much reliability literature to go along with "stress." When $F(\Upsilon)$ is substituted for $P(D|H,\Upsilon)$ in Equation 2, the formulation is sometimes referred to in the reliability literature as a "stress-strength model." Alternatively, particularly in the field of structural reliability, the random variables might be called "resistances" and paired with "loads" in "load-resistance models." In this paper the conjugate terms are damage potentials and damage strengths. In an experiment, we say damage will result (by definition) if the random strength is less than or equal to the damage potential level for that experiment. The probability of this happening is by definition a (cumulative) probability distribution function.

The concepts of damage potentials, damage indices, and damage strengths permit us to link the occurrence of a damage response modeled as a discrete event with the accompanying physical process described in terms of the information H and U_M. For example, physical theories concerning the behavior of materials are often based on a conceptual view of damage as a continuous evolutionary process that involves the distortion of the atomic and molecular matrices, the development of bond failures and

dislocations, and the formation of microcracks and microvoids of various types which link, coalesce, and eventually result in a macroscopically recognizable condition or event defined as crack initiation. Suitable damage indices for predicting initiation might be a stress-modified strain or a void volume fraction as formulated by some suitable, but idealized, model. In these cases damage strengths would be imagined to be critical values of strain or void volume fraction. If the stress and strain conditions are sufficient, the damaged region might continue to develop until it achieves a state referred to as plate penetration. A damage index useful for prediction of the plate penetration event might be the depth of the crack as formulated by a suitable, but idealized, model. The notion of critical crack depths as the conjugate strengths could be invoked to explain the test data. Obviously, the center of the strength distribution would fall somewhere around the plating thickness if the model is reasonable. Finally, the event of crack penetration through the plating thickness begins the process of through-crack extension. This is a continuous response that could be statistically modeled in a manner analogous to that used in our coupon tests example.

In these examples damage indices associated with crack initiation and penetration events were based on physical models of the continuous underlying processes. They can also be based on other considerations. Under the statistical approach, physical response theories are used (1) to combine or pool test data gathered under different conditions and (2) for predicting or inferring the outcomes of experiments conducted at untested conditions. Conceptually, the roles of models based on physical response theories are identical to the roles of the models employed in the simple linear regression analysis examples discussed earlier, which permitted us to combine data subpopulations labeled by different values of the independent variable and to infer behavior under untested conditions. Thus, from the viewpoint of statistical modeling, the principal role of the physical response theories lies in the formation of the model hypothesis.

In practice, when predicting damage defined as a discrete event, the formation of the model hypothesis involves two initial steps. First, we choose any function $\Upsilon(H,U_M)$ that we believe has the properties of a damage potential [i.e., constant Υ implies constant probability, zero Υ implies zero probability, and increasing Υ implies increasing (or constant) probability]. Second, we choose any appropriate family of strength distribution functions (e.g, the family of Weibull distributions is often applied to problems of structural damage). Let us denote this family as $F[\Upsilon;\alpha,\beta,\gamma]$, where α, β, and γ are undetermined parameters (the semicolon is used to separate them from

the value of the random variable). Then, combining F and Υ, we hypothesize the mean structure underlying the tests to be of the form $P(D|H,U_M) = F[\Upsilon(H,U_M);\alpha,\beta,\gamma]$. Using statistical estimation techniques, we select values of α, β, and γ that best fit the data. The fitted model is then subjected to a goodness-of-fit test to finalize the statement of the model hypothesis. The Bernoulli distribution, which governs the binary random response (D or not D), is fully characterized by a single parameter, its mean $P(D|H,U_M)$. Our estimate of the $P(D|H,U_M)$ function, obtained by substituting the estimated values of α, β, and γ into $F[\Upsilon(H,U_M);\alpha,\beta,\gamma]$, thus completes the estimation of the probability model for the full population of responses of interest.

Modeling Multiple Discrete and Continuous Response Variables:

Up to this point we have discussed statistical approaches for modeling single continuous responses and single discrete responses. Explosive tests of submarine hull models, of course, produce a continuum of responses across the structure that interact in complex ways to produce a final pattern of damage; in some cases the responses include hull rupture. Deterministic engineering models have been and can be developed to describe many of these responses and their complex interactions. We have seen how such models might be used to model the mean structures for both single continuous and single discrete nondeterministic responses. A response like the rupture of an externally stiffened submarine pressure hull, however, depends upon sequences of interacting responses leading up to rupture that are themselves quite difficult to model deterministically. Some of these are perhaps best regarded as discrete events (the initiation and penetration of cracks through one or more stiffening rings) and others are in the nature of continuous processes (stiffener buckling and crack extension).

Much of the uncertainty of the hull rupture response is a consequence of the uncertainties of the responses that precede hull rupture. The smaller (perhaps 5 to 10 percent) uncertainties associated with the initiations of fractures at individual stiffener weldment sites, for example, combine together to produce the larger uncertainties observed for hull rupture. Consequently, the difficulty of accurately modeling the hull rupture response deterministically is also compounded. For such problems it appears to be reasonable to construct multivariate statistical models (i.e., models involving more than one random response variable) and to use the multivariate response data

produced in each test (e.g., the locations and lengths of cracks that make up the fracture patterns) to estimate the parameters of such models. This is what we have attempted to do in the Hull Rupture Modeling Task of the SDM Project.

When a single test produces many discrete and continuous response outcomes of interest, the probability model for the responses is characterized by a joint distribution. In analogy with the case of a single response variable, the joint distribution describes how the vector of response variables varies randomly from one test to another. Population characteristics of the joint distribution include, as before, means, variances, higher order moments, etc. of the individual random outcomes, but in addition to these they also include correlations, covariances, etc. that describe the interactions and interdependencies of the responses. When the database of interest includes many unique sets of test conditions (as in the submarine hull rupture database), the mean, covariance, and other structures underlying the multivariate probability model may be quite complex functions of the modeling variables. The situation is similar to that associated with single-variate models, but there are now more population characteristics that must be modeled.

The goal of a statistical analysis of multivariate data is the same as that for single-variate data: to estimate the functions describing the population characteristics and, from these, to estimate the probability model as a whole. Again, deterministic models may be used to describe the means. The new problem that comes with multivariate data is that of modeling and estimating the interaction characteristics. Interactions are readily apparent in externally stiffened submarine hull response data. For example, if a fracture develops in a stiffening ring, the likelihood of a subsequent fracture occurring in the pressure hull in the immediate vicinity of the detached ring is considerably reduced, but the probability of a fracture in a more remotely located stiffener may be increased. This is because the ring detachment relieves the stresses built up in the hull locally, but may transfer the load to the other stiffeners. If the fracture first occurs in the hull plate at the base of a stiffener, the likelihood of a subsequent ring fracture may be reduced.

The interaction modeling problem can be simplified by recognizing that structural responses are local in nature. That is, a material element of a structure sees only the local stress and strain environment directly experienced by the element. The interaction of a local response with other responses occurring elsewhere in the structure is communicated through the stress and strain fields. If it is possible to track these local stress and strain conditions as they are affected by remotely occurring damage

processes, then it will be possible to model the local responses as if they were stochastically independent of the remote responses. This notion is referred to in the statistics literature as conditional independence.[8] Basically, conditional independence means that if we can deterministically model the loading transfer mechanisms that give rise to the dependencies among the responses, then the randomness of the various interacting responses can be described by set of single-variable distribution functions that are conditional upon the calculated local conditions. Then, instead of being concerned with a joint distribution function, we need only be concerned with simpler single-variate distribution functions because we have created a physical model for the interactions.

Deterministic structural models are reasonably accurate for tracking local responses except when the responses become sensitive to details not included in the models. If sensitive responses, such as buckling and fracture onsets, are modeled as discrete statistical events, deterministic models can be used to link such events to provide a description over the full range of the responses. This is the idea behind the Hull Rupture Modeling Task.

Summary:

In summary and more concisely, the statistical approach regards all responses as nondeterministic. The objective is to use response data to describe the probability model underlying the responses; this is the (joint) distribution function for the random response outcomes. Specification of the probability model entails the specification of characteristics of the response distribution (i.e., means, covariances, etc.) throughout the domain of the modeling variables. Deterministic models based on the physics of the responses are useful for these specification purposes. It is convenient to model some responses, such as threshold phenomena, as discrete events, particularly when the physical mechanisms underlying the responses cannot be completely specified. Discrete responses introduce the notions of random strengths and strength distributions. Observations of continuous responses, which can be approximated using deterministic models, are associated with error distributions. The strength distributions and error distributions describe the random aspects of the probability model, while the characteristics functions of the response distribution describe the systematic aspects of the probability model. When multiple interacting responses are

of interest and when physical models for the interaction mechanisms are possible, conditional independence of the responses can be used to construct a probability model in which response dependencies are adequately represented. A class of possible probability models is first developed from which a best candidate is selected by a statistical estimation procedure; this forms the model hypothesis. The estimated model hypothesis is rejected if it fails statistical tests that are sensitive to the differences between the response behavior hypothesized by the model and the actual response observations. A model that is not rejected can be used to make predictions of behavior means, variances, and other distribution characteristics for arbitrary conditions and to construct decision models .

SDM PROJECT

The technical objective of the Hull Rupture Modeling Task is to develop a stochastic simulation model for multivariate failure responses of interest in the database consisting of explosively tested stiffened shells. The task plan is to build the simulation model by employing well-founded deterministic physical models to describe the mean structure of the response outcomes. Selected continuous responses simulated by the model, namely crack lengths, will be affected by randomly assigned errors, and related onset phenomena, namely fracture initiation and crack penetration through the plating thickness, will be affected by randomly assigned strengths. The simulation model outcomes, thus, will depend upon strength and error distribution functions and their associated parameters. The parameter estimation problem is to fit the simulation model to the data so that the fitted simulator generates responses with systematic characteristics like those of the database. To accurately capture the interactions between the responses, the simulation model must also include the loading transfer mechanisms responsible for the dependencies in the structural model. The adequacy of the various modeling assumptions will be tested by one or more suitably designed statistical hypothesis tests. The fitted simulation model will then be used to estimate response probabilities of interest for arbitrary structural designs and problem conditions to which the model can be interpolated or extrapolated with reasonable accuracy.

The specific test outcomes selected for modeling, namely the occurrences of crack initiation and plating penetration and the final lengths of cracks, were chosen because (1) they are rather easily extracted from the Navy database, (2) they are damage states of considerable interest, and (3) the lengths of stiffener cracks, in particular, affect the overall compliance of the structure and the development of cracks at neighboring sites. By pinning responses to these data, we hope to obtain a more reliable model for predicting hull rupture than models based upon small specimen data.

Additional data could, in principle, also be explicitly included in the statistical model. In particular, we could include data on shell deformation and stiffener

buckling. It was decided, however, to develop and test the deformation and buckling (also called tripping) algorithms of the model off-line, so to speak, rather than model the random aspects of the deformation and fracture data simultaneously. This was to avoid over-complicating an already complex statistical model. Further, since fracture responses are closely linked with deformation, the test of the model hypothesis against the fracture data would be an indirect test of the deformation algorithms. Moreover, the fracture responses were regarded as the responses of primary interest.

Our modeling strategy was to first develop models for hull deformation, crack initiation, plate penetration, and crack extension that would permit us to simulate the development of fracture patterns for a given description of the explosion output. The computer model for these combined responses is referred to as the response generating algorithm or RGA. Basically, the RGA functions like a numerical experiment. A flow chart of the RGA appears in Figure 6. While we could, in principle, use one of the existing computational mechanics codes as the core of our RGA, such codes are without exception too slow at the present time on typical machines to meet our modeling requirements.* Hence, we have undertaken the development of faster, sufficiently accurate structural models. (We include a discussion of what is meant by "sufficiently accurate models" in the following section.) This work has been a joint collaborative effort of NSWC, the Massachusetts Institute of Technology (MIT), and SRI International.

To achieve rapid computation speeds, MIT and NSWC have pursued the development and coding of highly analytical models for the deformation of arbitrary externally ring-stiffened cylinders with and without the presence of cracks.[9-13] The MIT modeling approach is based on a statement of dynamic equilibrium which equates the rate at which energy is dissipated by plastic work and fracture to the rate at which energy is supplied to the shell by the explosion. The development of equations of motion from this is aided by making various simplifying assumptions regarding the behavior of the shell and the ductile shell material. These include the assumptions that (1) the dominant response mechanisms of the shell are axial stretching and circumferential bending, (2) the material behavior can be approximated as rigid perfectly plastic with simple yield criteria, and (3) the stiffening rings respond by

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* Such computational mechanics models are currently being developed by ONR under the Modeling and Simulation Project. We foresee their eventual use in statistical applications as discussed here when high-speed computational technology becomes widely available.

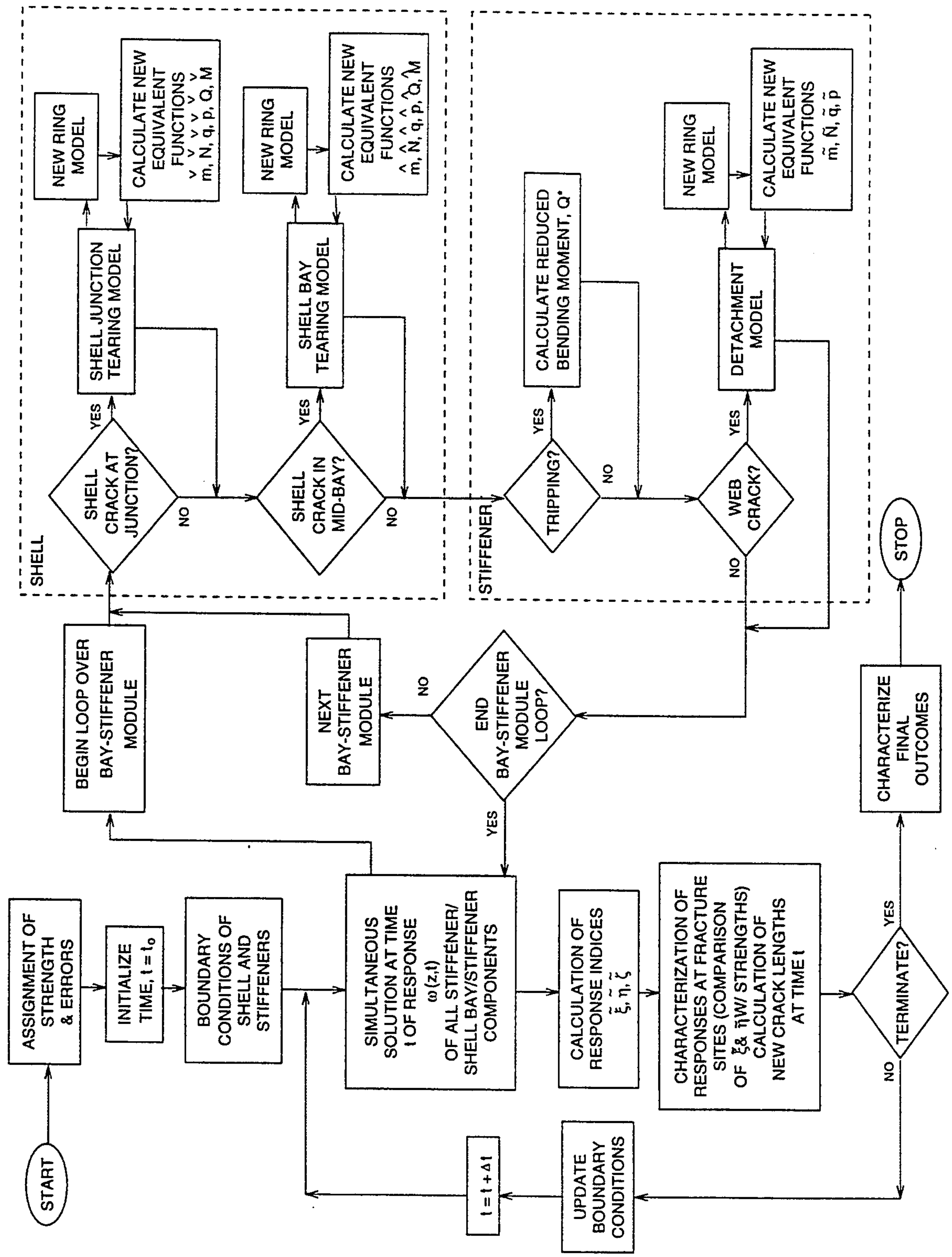

FIGURE 6. RESPONSE GENERATING ALGORITHM

circumferential bending and out of plane buckling with prescribed geometries and buckling patterns. In addition, a relatively simple model for crack growth along prescribed paths is assumed. These assumptions permit the development of reasonable equations of motion that are solved both analytically and numerically for the deformation of the pressure hull and the growth of stiffener and shell cracks (that, as will be seen, have come into existence by stochastic strain-driven initiation and penetration processes). The equations are regarded as being most accurate directly opposite the explosive charge along the longitudinal centerline of the load.

The development of crack initiation and growth models has been the primary responsibility of SRI International. Hull and ring fractures tend to occur within or near the heat-affected zones associated with the ring-to-shell weld joints (T-joints). Experimental and analytical studies carried out at SRI showed that the variations of material properties associated with weldments of different sizes and weldments made by different welding processes can cause significant differences in weldment fracture behaviors. Since we are using the reduced-scale models of our database which differ in size as well as in other weldment features from the full-scale pressure hulls of interest, it is important to model the systematic changes of behavior associated with these differences. It is also known that fracture growth processes in pure single-material, geometrically similar specimens are affected by changes of scale; this is sometimes referred to as the fracture size effect.

To guide the fracture modeling effort, SRI International has developed DYNA3D finite element models of two-dimensional T-joint weldments (plane strain is assumed along the lengths of the welds).[14] Each model is finely meshed using small elements with multiple, tied, corner nodes in the fracture region near the stress concentration at the toe of the weld. A local fracture process is assumed and simulated by applying a material-dependent, semi-empirical fracture criterion to the elements. When the criterion is met over a multiple-element region of fixed size referred to as the process zone, the tied nodes at the center of the zone are released and allowed to move independently. Propagation of the crack is thus simulated as the process zone at the crack tip moves sequentially through the thickness of the plating along a path determined by the material properties and the local stress and strain fields.

The SRI fracture code has been shown to capture the systematic fracture behavior observed for both statically and dynamically tested, welded T-joint specimens.[15-16] And it is thought to model the various mechanisms described above to

a reasonable level of accuracy. Currently, its ability to predict the fracture size effect is being further evaluated by comparing it with the results of tests of precision nonwelded specimens of different sizes. The code is also being used to study the results of (mixed-mode) tension-torsion tests that will give an indication of its ability to predict the crack extension or tearing mode. A full three-dimensional model of a single stiffening ring on a two-frame-width cylinder has been developed to provide guidance to MIT's effort to model crack extension. Other similar configurations of interest are also being studied. Finally, the code is being used to develop simple, and approximate, strain- or energy-based analytical fracture initiation indices and through-thickness crack penetration indices that can be calculated using the approximate MIT deformation model. These will be used in the statistical code to model the processes of crack initiation and penetration as discrete random events. Based on preliminary work, physically motivated and relatively simple functions of the longitudinal strains and curvatures calculated by the rigid-plastic model appear to be reasonable choices for both the initiation indices and the penetration indices.

The goal of the statistical modeling effort, as stated previously, is to estimate the probability structure of a population of tests represented by the Navy stiffened shell database and to use this estimated structure to predict the fracture responses of full-scale submarine pressure hulls. Said differently, we intend to fit the RGA to the database so that it can be used to predict the failure of full-scale submarines. To accomplish this, the data for each test are organized according a practical discretization scheme. Each structure is divided into annular regions called probability elements as indicated in Figure 7. The elements represent a sufficient resolution of the probability field over the shell and they identify regions of expected crack formation. In particular, we are concerned with (1) cracks in the stiffener web plates that, upon extension, separate the rings from the shell, and (2) cracks in the shell near the stress concentrations associated with the stiffener-to-shell weldments. As shown in Figure 7, the three types of probability elements are called web elements, junction elements, and bay elements.

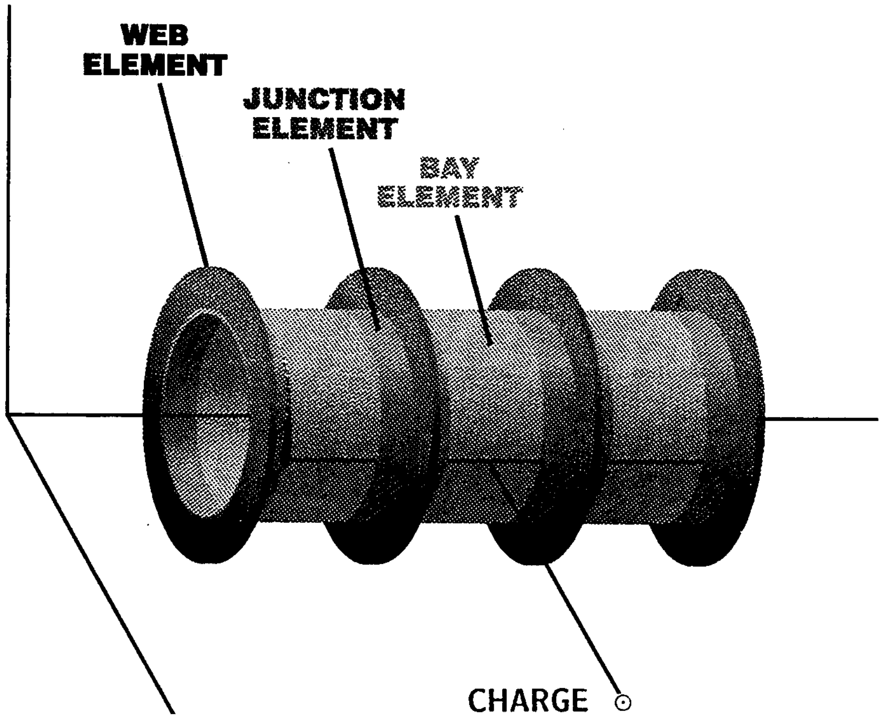

FIGURE 7. PROBABILITY ELEMENTS FOR STIFFENED SHELL ANALYSIS

From experiments we know that within each probability element crack initiation tends to occur at the point of maximum (triaxial-stress modified) strain*; we call this single point within each element the element "event site." The locations of the event sites for the different elements all lie within the loading symmetry plane as shown in Figure 8. After each RGA time step, fracture initiation indices are monitored at each event site and compared with initiation strengths that were initialized at the beginning of the run by random draws from the initiation strength distribution. When a strength has been met or exceeded at an event site, the fracture penetration index is then calculated and compared with its (also randomly assigned) fracture penetration strength. Crack extension is started when the penetration index equals or exceeds the penetration strength. The length of the crack is determined at any given time by the crack extension algorithm; the calculated value is referred to as the crack extension index. The crack length actually used in the next time step is

* We view such conditions as the rotational and dilatational nature of the stress field (triaxiality) as affecting the damage per unit strain increment and the critical damage strengths.

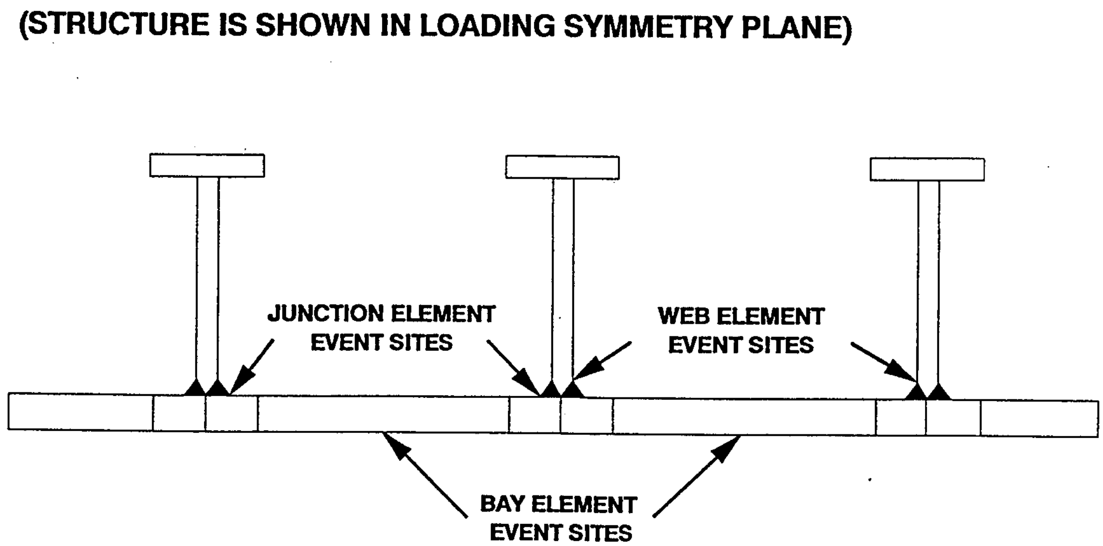

FIGURE 8. PROBABILITY ELEMENT EVENT SITES

obtained by multiplying the extension index by a multiplicative error* that is randomly drawn from an error distribution and retained for that event site for the duration of the run.

A random fracture pattern is thus produced at the end of each RGA run that depends upon the random strengths and errors drawn for the run. Probabilities of fractures thus depend upon the fitting parameters associated with the strength and error distributions. Other fitting parameters could be added elsewhere within the RGA at the discretion of the modeler. Values which maximize the likelihood function† associated with the database are assigned to all fitting parameters. The

* Multiplicative errors are used so that error standard deviations will be proportional to mean crack lengths. This assumption for the underlying structure of the standard deviations is based upon informal experimental observations. The assumption will be later tested statistically.

† The likelihood function is defined as the probability model (the model for the joint frequency function of the observed responses) evaluated at the values of the random outcome variables observed in the database. It is a function of the model parameters. For discrete outcomes the likelihood function can be thought of as the probability of the observed sample under the model. The maximum likelihood estimation technique is discussed in most textbooks on mathematical statistics.

likelihood function is represented as an integral which can be calculated by using an efficient form of Monte Carlo integration. Reference 17 discusses such a method that can be used when the sequence of site fractures in each test is known. It remains to generalize this model so that it can be used to analyze fracture data for the more usual case where the fracture sequences are unknown. It is believed that a goodness-of-fit test for the fitted model can be developed around the concept of likelihood ratio tests. Finally, damage probability predictions for arbitrary conditions will be made by using efficient Monte Carlo estimators obtained in a manner similar to that by which the likelihood function is calculated.

As implied earlier, the modeling techniques developed for application to the Hull Rupture Modeling Task are quite general and can be extended to the Catastrophic Failure Task and the Concept Assessment Methodology Task. The latter, in particular, will require the use of additional statistical concepts such as loss functions or utility functions, which derive from statistical decision theory, to manage notions of consequence and importance. Our purpose in doing so is to more effectively address the needs of the Navy described earlier.

CONCLUDING DISCUSSION

Sophisticated structural response codes are required in a statistical analysis when the mean of the probability model has a complex structure. They are also useful for modeling covariance structures associated with response interactions. Clearly, both mean and covariance structures become more complex when we wish to control more variables. And we are motivated to control more variables because we would like our model to be applicable to a wide variety of problem descriptions or conditions. We would like be able to use our submarine response model to describe not just the responses of one submarine design, but the responses of submarines of many shapes, sizes, and construction details.

The quasi-deterministic approach to this problem can be viewed as an attempt to extrapolate the results of small specimen tests (upon which the constitutive and failure models are based) to predict the responses of full-scale submarines. The statistical approach is an attempt to extrapolate the results of existing reduced-scale submarine model tests to achieve the same end. The statistical approach attempts to control far fewer variables than the quasi-deterministic approach. Variables that are not controlled include the deviations from the idealized hull design geometries, geometrical and material imperfections associated with weldments, residual strains, explosion variabilities, and a host of other quantities that are permitted to vary in the explosion damage tests in natural ways characteristic of the subpopulations of interest. Consequently, the uncertainties do not have to be explicitly accounted for in the final analysis when system reliability is assessed. Under the statistical approach many uncertainties are already accounted for in the test data. Under the quasi-deterministic approach many are not, and this necessitates an expensive post-computational analysis of the uncertainties. Under both approaches enough variables are controlled to permit predictions to be made for a wide variety of test conditions and structures of interest.

Model uncertainty is a statistically testable and formally defined proposition under the statistical approach. It can be shown to be closely tied to the notion of estimator consistency, i.e., convergence of the estimated quantities to the underlying population characteristics as more empirical information is acquired. Model

consistency should, in the author's opinion, be required of all modeling techniques. That is, any proposed model of the systematic behavior associated with a population should improve as more empirical information is acquired from the population. Statistical modeling techniques and estimator functions are almost always designed to satisfy this basic requirement.

Consistency of the estimated population characteristics and the probability model as a whole is forced by the goodness-of-fit test and by other tests of residuals. These tests can be thought of as strategies for controlling the systematic modeling errors. In effect, the tests make the experimental data the arbiter of the level of approximation that is sufficient and acceptable in modeling the physical processes. To make this point a well-known statistician once facetiously proposed calling the goodness-of-fit test the "goodenoughness-of-fit" test,[18] meaning, that if the estimated probability model is not rejected by the goodness-of-fit test, it is good enough for making predictions. This should be qualified by the stipulation that accuracy bounds for the predictions that are associated with the random errors also be reported.

There is risk associated with the use of any response prediction model. The statistical approach affords a way of quantifying that risk and taking it into account in subsequent decision making. The statistical approach can be used to explicitly show how experimental information can be used to reduce decision risk and how to best design experiments so that reduction of risk is optimized. The approach can also be used to quantify the level of risk associated with an existing prediction capability to show if additional experimental data are actually needed. The notion that models can be validated, once and for all, and used risk-free under all circumstances is an obvious oversimplification. Model adequacy involves many issues, and serious consideration of model risk requires that the consequences and utilities of actions be taken into account. The statistical approach provides an essential framework for conducting such examinations.

In this discussion of the statistical approach we have presumed that the data being modeled were representative of the population of interest. If the data are contaminated by unwanted sources of variation and the tests are expensive (and cannot be repeated), it is often possible, and useful, to model the contamination along with the process of interest to extract the information of value; that is, to construct a probability model for the contaminated data. If the tests can be repeated, then repeating them in a more precise fashion so that they (can be believed to) truly represent the the population of interest is perhaps worthwhile.

We have sought to emphasize the many benefits of the statistical approach. As a synthesis and extension of the semi-empirical and quasi-deterministic approaches it brings additional modeling capabilities without sacrificing the predictive power associated with the quasi-deterministic method when this power is required for the solution of problems. Statistical methods are still quite new to many engineering communities that might benefit by using them. Statistical modeling is still too often thought of in terms of off-the-shelf canned programs that are rather blindly applied to data, rather than in terms of concepts and principles to be used in conjunction with physical principles to model the phenomena. At the same time, many of the techniques widely used by engineers to model physical processes are still unfamiliar within many statistical and mathematical circles. The hull rupture prediction problem is only one of many interdisciplinary problems that lie in the cracks between these disciplines.

Since all responses are nondeterministic, if one measures them closely enough, statistical modeling techniques are fundamental to describing system behavior and should be a part of the analyst's armamentarium for problem solving. The reason why existing statistical models often do not seem relevant to the solution of realistic engineering problems is perhaps due to the different roots from which the two sciences have sprung. For example, much of statistical theory has been developed for problems occurring in the biological and social sciences. In these areas the systems being studied are so complex and the intrinsic uncertainties of the responses so involved that little is known of the governing mechanisms operating within the systems. Consequently, the probability models employed, such as those found in the theory of linear models, are necessarily simple -- involving simple relationships between the modeling variables. Thus, these disciplines may be characterized as having relatively weak models for the systematic behavior and relatively strong models for the random behavior (consider, for example, analysis of variance theory). On the other hand, by design, engineering systems operate according to well-understood physical principles, which enables the systematic behaviors to be described over a wide range of conditions. Random behavior that is in evidence when engineering systems fail has not received commensurate treatment. While much of the statistical theory required for such applications has already been developed, much modeling and theory development also remains to be accomplished.

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